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MARKETS WITH PREPAYMENTS


by

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Thesis submitted for the degree of PhD

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To the memory of my father



Summary

The purpose of this thesis is to examine the behaviour of consumers and producers in markets where the consumers of a product have to *prepay* for their purchases and then wait for a period of time before the goods are delivered to them.

Chapter 2 examines the role of prepayments as a means to support efficient exchange. We develop a framework to discuss contractual arrangements between a buyer, who places an order for a product, and a seller, who makes an investment in specific assets in response to the buyer's order. We find that contracts stipulating a non-refundable prepayment which is to be paid when the order is placed can support efficient exchange as long as the prepayment is set equal to the amount of the investment in specific assets.

In chapter 3 we develop a model of a market where some customers of a firm with monopoly power must pay an advance deposit for the right to purchase a new durable good sooner than others. We will show that this behaviour can be explained by a model in which a monopolist, having already spent money on R & D but being uncertain of the profitability of his product due to cost uncertainty, charges the non-refundable prepayment in order to be able to recover the R & D cost in the case where the product turns out to be unprofitable and, therefore, is not produced. Some high-valuation customers will be prepared to bear the risk of losing the prepayment (in the case where the product is not produced) as long as they are given priority over others in the delivery of the product.

Chapter 4 examines the role of prepayments as an integral part of the pricing strategy of an incumbent firm who is privately informed as to the level of cost and is concerned with deterring entry in a market of a new product. We show that there exist separating equilibria in which the low-cost incumbent's pricing policy is successful in deterring entry. This pricing policy involves the signing (by the incumbent and the buyers) of a number of contracts one period before production starts. These contracts require consumers who have signed them to prepay for their purchases of the product before production starts and take delivery after the end of the production period.

Chapter 5 extends the model of chapter 4 to allow for heterogeneous consumers and the possibility for the incumbent to choose between two patterns for serving consumers who are willing to prepay. Both serving patterns give rise to a unique separating equilibrium in which prepayments prevent entry. In addition, rationing of those consumers who are willing to prepay may emerge in equilibrium and this depends on the serving pattern that is adopted by the incumbent.

In chapter 6 we show that an incumbent seller with a known unit cost can reduce the probability of entry into his market of an entrant who faces a fixed cost of entry and whose variable unit cost is unknown by signing contracts requiring buyers to pay for the product before it is delivered to them. Buyers may sign the contracts even though they know that the entrant may turn out to be more efficient than the incumbent. This behaviour is mainly due to the assumption made that each buyer's reservation price is decreasing in the number of buyers who purchase the product.

Table of Contents

Summary	iii
List of Figures	xi
List of Tables	xiv
Acknowledgements	xv
 PART 1 - INTRODUCTION TO THE ANALYSIS	
<i>Chapter 1 - Introduction</i>	2
 PART 2 - ANALYSIS OF PREPAYMENTS	
 <i>Chapter 2 - Contractual Exchange or Vertical Integration? The Role of Prepayments in Supporting Efficient Contractual Exchange</i>	
1. Introduction	14
1.1. Contractual Exchange: Some Introductory Remarks	16
1.2. Contractual Exchange When the Seller Values the Prepayment Less than the Buyer	17
2. Reputation Effects and Contract-Handling Agents: Why the Seller May Value the Prepayment Less than the Buyer	19
3. The Model	21
3.1. Technologies and Costs	21
3.2. Demand and the Contracting Process	21

4. Market Exchange <i>m</i>	22
5. Vertical Integration	24
6. Contracting Scheme <i>p</i>	24
7. Contracting Scheme <i>p</i> When the Supplier Values the Prepayment Less than the Buyer	29
7.1. Contracting Scheme <i>p1</i>	31
7.2. Contracting Scheme <i>p2</i>	35
7.3. Contracting Scheme <i>p3</i>	37
7.4. Contracting Scheme <i>p4</i>	40
8. Prepayments as Opposed to Hostages: Some Critical Remarks on Williamson's Hostages as a Means to Support Exchange	43
9. Concluding Remarks	46
Appendix	47
Notes	50

Chapter 3 - An Optimal Sales Strategy for a Monopolist of a New

Durable Good: Contracts and Prepayments

1. Introduction	60
2. Previous Relevant Research	62
3. The Period-2 Price and the Coase Conjecture	66
4. Consumers' Behaviour and the Expected Profit-Maximizing Sales Strategy	67
4.1. Consumers' Behaviour	68
4.1.1. Determination of the Marginal Consumer, <i>T</i> , Given	

a Period-0 Contract $\{p_0, p_1, p_2\}$	72
4.1.2. Effects of Changes in p_0 and γ on the Value of the Marginal Consumer	74
4.2. Determination of the Expected Profit-Maximizing Pricing Policy Given the Equation for the Marginal Consumer T	75
5. Welfare Implications of the Expected Profit-Maximizing Pricing Policy	82
5.1. Expected Welfare When the Monopolist's Pricing Policy Involves Prepayments	83
5.1.1. Case 1: $c(T^*) < c_R \leq p_1^*$	83
5.1.2. Case 2: $0 \leq c_R \leq c(T^*)$	84
5.1.3. Case 3: $p_1^* < c_R \leq 1$	85
5.2. Expected Welfare in a First-Best Environment, EW^{FB}	86
5.3. Expected Profits and Welfare in the No-Prepayments Monopolistic Setting	87
5.3.1. Case 1: $c_R < p_1/\gamma$	89
5.3.1.1. Consumers' and Monopolist's Behaviour	89
5.3.1.2. Ex Post Welfare	90
5.3.2. Case 2: $c_R > p_1/\gamma$	91
5.3.2.1. Consumers' and Monopolist's Behaviour	91
5.3.2.2. Ex Post Welfare	92
5.3.3. Case 3: $c_R = p_1/\gamma$	92
5.3.3.1. Consumers' and Monopolist's Behaviour	92
5.3.3.2. Ex Post Welfare	93

5.3.4. Expected Profits and Expected Welfare	93
5.4. Expected Welfare Comparisons	94
6. The Prepayment as a Facilitating Device to Secure a Line of Credit	96
7. Conclusion	97
Notes	98

Chapter 4 - Prepayments and Contracts as Signals of Cost

1. Introduction	105
2. Description of the Model	107
3. Review of Relevant Research	109
4. The Model	110
5. Sequential Separating Equilibria	113
5.1. Sequential Rationality for the Low-Cost Incumbent	113
5.2. Sequential Rationality for the High-Cost Incumbent	114
5.3. Sequential Separating Equilibria	115
6. Undominated Sequential Separating Equilibria	117
7. Pooling Equilibria	121
8. Concluding Remarks	122
Appendix	124
Notes	127

Chapter 5 - Prepayments and Contracts as Signals of Cost in a Model With Heterogeneous Buyers

1. Introduction and Summary of the Model	133
2. Review of Previous Research	135
3. Main Elements of the Model	137
4. The Efficient-Serving Scheme	138
4.1. Description of the Efficient-Serving Scheme	138
4.2. Sequential Separating Equilibria	140
4.2.1. Sequential Rationality for the Low-Cost Type	140
4.2.2. Sequential Rationality for the High-Cost Type	142
4.3. Undominated Sequential Separating Equilibria	145
4.4. Description of the Result and Intuition	146
5. The Proportional-Serving Scheme	149
5.1. Description of the Proportional-Serving Scheme	149
5.2. Sequential Separating Equilibria	150
5.2.1. Sequential Rationality for the Low-Cost Incumbent	150
5.2.2. Sequential Rationality for the High-Cost Incumbent	152
5.4. Description of the Result and Intuition	155
6. Equilibrium Effects of Changes in c_s	156
6.1. Effects on the Size of Equilibrium Prepayments, p_i^E and p_i^P	156
6.2. Effects on Equilibrium Profits Under Both Serving Schemes	157
7. Conclusion	159
Appendix	161
Notes	174

Chapter 6 - Prepayments as a Barrier to Entry

1. Introduction	193
2. Description of the Model	196
2.1. The Incumbent's Ability to Reduce the Size of the Market in the Post-Entry Game	197
2.2. The Incumbent's Incentives to Reduce the Size of the Market in the Post-Entry Game	198
2.3. The Size of the Post-Entry Market and the Probability of Entry	198
2.4. Efficiency and Ex Ante Welfare	198
3. The Model	199
3.1. Basic Elements of the Model	199
3.2. Decision Sequence and Information Structure	201
3.3. The Size of the Market in the Post-Entry Game and the Probability of Entry	202
3.4. Equilibrium Outcomes	204
3.4.1. Case 1: Only Buyer 1 Prepays or $n = 1$	205
3.4.2. Case 2: Both Buyers Prepay or $n = 0$	208
3.4.3. Case 3: No Buyer Prepays or $n = 2$	210
4. Expected Profits Versus Expected Consumers Surplus	212
5. $n = 1$ Versus $n = 0$: Relative Profitability of Prepayments	213
6. Welfare Effects of Prepayments	214
7. Conclusion	215
Appendix	218
Notes	224

PART 3 - CONCLUSION

Chapter 7 - Conclusion

228

References

232

List of Figures

Chapter 2

Figure 1 - Cost Distinctions	54
Figure 2 - Demand Distribution and Costs of Supply	55
Figure 3 - The Relationship Between the Equilibrium Contract- $\rho\beta$ Payment and Total Costs, and Between the Equilibrium Contract- $\rho\beta$ Prepayment and the Specific-Asset Investment, Given a Value for α	56
Figure 4 - The Equilibrium Contract- $\rho\beta$ Full Payment as a Function of v , k and α	57
Figure 5 - The Equilibrium Contract- $\rho\beta$ Prepayment as a Function of v , k and α	58

Chapter 3

Figure 1 - The Marginal Consumer T Given the Period-0 Contract (p_0, p_1, p_2)	99
Figure 2 - Effects of Changes in p_0 and γ on the Marginal Consumer	100
Figure 3 - The Expected Profit-Maximizing Solution	101
Figure 4 - Effects of Changes in γ on T^* and p_0^*	102
Figure 5 - Cost Realization and the Consumers' Decision in the No-Prepayments Monopolistic Regime	103
Figure 6 - The Period-1 Kinked Demand Curve When the Monopolist Does Not Use Prepayments	104

Chapter 4

Figure 1 - Sequential Rationality for the Low-Cost Type	129
Figure 2 - Sequential Rationality for the High-Cost Type	130
Figure 3 - Sequential Separating Equilibria	131
Figure 4 - Undominated Sequential Separating Equilibria	132

Chapter 5

Figure 1 - The Efficient-Serving Scheme	177
Figure 2 - Sequential Rationality for the Low-Cost Type Under the Efficient-Serving Scheme	178
Figure 3 - Sequential Rationality for the Low-Cost Type Under the Efficient-Serving Scheme in the Case Where $c_s \in [0.5, 1]$	179
Figure 4 - Sequential Rationality for the High-Cost Type Under the Efficient-Serving Scheme	180
Figure 5 - Sequential Separating Equilibria Under* the Efficient- Serving Scheme	181
Figure 6 - Sequential Separating Equilibria Under the Efficient- Serving Scheme When c_s is Much Bigger than c_l	182
Figure 7 - The Unique Undominated Sequential Separating Equilibrium Under the Efficient-Serving Scheme	183
Figure 8 - Optimality of the Unique Undominated Sequential Separating Equilibrium Under the Efficient-Serving Scheme	184
Figure 9 - The Proportional-Serving Scheme	185

Figure 10 - Sequential Rationality for the Low-Cost Type Under the Proportional-Serving Scheme	186
Figure 11 - Sequential Rationality for the High-Cost Type Under the Proportional-Serving Scheme	187
Figure 12 - Sequential Separating Equilibria Under the Proportional- Serving Scheme	188
Figure 13 - The Unique Undominated Sequential Separating Equilibrium Under the Proportional-Serving Scheme	189
Figure 14 - Equilibrium Effects of Changes in c_h on $p_i^P - p_i^E$	190
Figure 15 - The Variation in Equilibrium Profits Under the Proportional-Serving Scheme Minus Equilibrium Profits Under the Efficient-Serving Scheme	191
Figure 16 - Decomposition of the Difference Between the Equilibrium Profits Under the Two Serving Schemes in Profits from Prepayments and Profits from Sales to the Residual Demand	192
 <i>Chapter 6</i>	
Figure 1 - The Extensive-Form Game Between Buyers 1 and 2	226

List of Tables

Chapter 2

Table 1 - Numerical Solutions for Contract $\rho /$ When $\nu = 0.33$
and $k = 0.15$

59

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PART I

INTRODUCTION TO THE ANALYSIS

1 INTRODUCTION

The purpose of this thesis is to offer an economic rationale for a widespread phenomenon: potential buyers of some products have to *prepay* for their purchases and wait for a period of time before the goods are dispatched to them. Although *prepayments* form an integral part of the pricing policy of a considerable number of firms, their presence has not been explained by a systematic theory in the economics literature. This thesis is an attempt to provide such a theory. Our aim is to investigate the circumstances that induce producers to use prepayments in a profitable way as well as to explore the implications of the use of prepayments for social welfare.

In real-life examples of business practices, the presence of prepayments can be indicated by the use of different names. *Down payments*, *up-front payments*, or *(advance) deposits* are some of them. Furthermore, prepayments can be either part of the total payment for one unit of the product or they can constitute the total payment themselves. In the former case, it is natural to adopt the position that prepayments form a *premium* over and above the unit price of the product while in the latter, one would expect that they take the form of *advance-purchase discounts*.

Prepayments are most likely to be part of the pricing strategy of firms producing to order either with the purpose of meeting each customer's particular wants or because producing the particular good is a time-consuming procedure which renders mass production impossible. The aircraft industry is an example which has a direct bearing on both previous points.

Nevertheless, the presence of prepayments may not necessarily be the outcome of the characteristics of the production side of the market only. It can also be dictated by the attitudes of consumers towards the attributes of the goods under question. To illustrate this point, we quote the following example from the *The Independent* newspaper (31/10/92):

...so eager were consumers to get their hands on the new 730i or 735i...that they were prepared to pay a *premium*. (Emphasis added).

In this example, some consumers are willing to pay a premium (over and above the unit price of the product) now in order to buy an advantageous place in the queue for the delivery of the forthcoming product. In effect, the situation involves a Veblen good so that early consumers obtain prestige but later ones do not.

Besides the durable-goods industry, prepayments can also be found in the services industry. Thomas (1991) reports on how to pick a private investigator:

...There should be a contractual written fee agreement with the investigator, and most reputable investigators will request a retainer fee and *prepayment* covering up to half the anticipated cost of the assignment in advance. (Emphasis added).

Charging a non-refundable prepayment, protects the investigator against the client's potential opportunistic behaviour. In the absence of any prepayment, the investigator would not be able to recover the set-up costs of the client's case if the client chose to cancel the contract with the investigator. Even if opportunistic behaviour is to be assumed away, there is still a reason why a *reputable* investigator may find it in his interest to charge prepayments. Suppose that the outcome of the investigation is uncertain and this is common knowledge. In spite of the risk (that the investigator's clients face) of losing the prepayment as a sequel of this

uncertainty, the investigator's good reputation for accomplishing his tasks can be a force that induces his clients to prepay. Hence, the investigator can still recover (part of) his set-up costs in the case where his client's case is not solved.

Another example is offered by Seal (1991):

Marriott Hotels, Resorts and Suites is offering 21-day and 14-day advance-purchase room rates at 192 hotels from June 21 through September 2. The payments are nonrefundable and deeply discounted. During its 1990 holiday test run, Marriott's nonrefundable advance-purchase program raised volume at 178 participating hotels by 10%, compared with the same period in 1989.

The obvious explanation for Marriott's strategy is that this strategy aims at attracting as many customers as possible during the highly competitive summer-time. A less obvious explanation is that this strategy reveals some information about Marriott's operational efficiency. If this is true so that customers are convinced that Marriott's strategy cannot be imitated by other hotels, then there will be a queue of customers waiting to book the limited number of rooms that are offered at the special discount rate. When no more such rooms are available, customers will have to pay the normal higher rate. This explanation is consistent with the fact that the advance-purchase payments are non-refundable. When customers are convinced that no other hotel will imitate Marriott's strategy in the period that will elapse between the booking of the room and checking in, they will be willing to pay a non-refundable prepayment.

The above examples of business practices provide us with the motivation to construct a theory that can explain the reasons that induce firms to use prepayments as part of their pricing policies. This theory is described in detail in the next five chapters of the thesis.

The first of these chapters examines the role of prepayments as a means to support efficient

exchange between a buyer who places an order for a product and a supplier who makes an investment in specific assets in response to the buyer's order. We will start by examining the market exchange between the buyer and the seller under conditions of *uncertainty* and *asset specificity*. Asset specificity refers to durable investments that are undertaken in support of particular transactions. The product in question is assumed to be produced by a special-purpose technology which requires such an investment. The opportunity cost of investments in transaction-specific durable assets is much lower in best alternative uses should the original transaction be prematurely terminated [Williamson (1985, p. 95)]. The presence of asset specificity would not cause any problems if uncertainty in the demand side of the exchange were assumed away. We assume, however, that demand is stochastic so that, although the buyer has already placed an order and the supplier has made the specific-asset investment, the buyer can either take or refuse delivery depending on his value of the product which is realized after the investment in specific assets has been made. If the realized value of the product is such that the buyer prefers to cancel the order and defect from the agreement under the specific-asset technology, the producer will be holding unproductive assets which cannot be economically shifted to other uses. The prospect of this loss will prompt the producer to make efficiency-distorting price adjustments.

In order to deal with the inefficiencies caused by the lack of *safeguards* in the market exchange between the buyer and the seller, we will examine two alternative ways of organizing transactions of above kind: *contractual exchange* and *vertical integration*. Vertical integration will yield the efficient outcome with which different contracting alternatives will be compared. These contracting schemes will share the characteristic of explicitly introducing safeguards against the buyer's potential opportunistic behaviour. The particular safeguard that we are interested in examining is *prepayments*: the buyer is charged an up-front payment

when he places the order. We will show that the prepayment will support an efficient outcome as long as it is set equal to the amount of investment in specific assets. The buyer will confirm the order and exchange will take place whenever the realized value of the product is greater than or equal to marginal cost of production. Under these circumstances, contractual exchange is preferable to vertical integration, taking into account the fact that internal organization of transactions is characterized by incentive and bureaucratic disabilities.

Nevertheless, prepayments will not support an efficient outcome if the seller values the prepayment less than the buyer. This will be the case where although the buyer pays a prepayment which is equal to the investment in specific assets (this will be referred to as the full value of the prepayment), the seller values the received prepayment at an amount which is smaller than that investment. The nature of the inefficiency will depend on (1) whether the full value of the prepayment is reduced by an amount proportional to that value or by a constant amount and (2) whether the full value of the prepayment is reduced under all possible demand realizations or under those realizations that induce the buyer to cancel the order. In this environment, internal organization of the transaction through vertical integration becomes a more appropriate mode of economic organization.

The example of the private investigator is a case that comes under the above theory. The investigator charges his clients prepayments in order to recover the set-up costs that are specific to each client's needs in the case where a particular client defects from the agreement. Most bilateral exchanges (ranging from every-day simple transactions to transactions that involve purchases of heavy durable goods) require the payment of prepayments whose presence can be explained by the theory that we have described above (and is presented in detail in chapter 2). There are, however, less straightforward reasons why prepayments exist. These are analyzed in the remaining chapters.

In chapter 3 we develop a model of a market where some customers of a firm with monopoly power must pay a prepayment for the right to buy a new durable good sooner than others. We consider a firm that has developed a new product which is going to be produced and launched into the market only if the unit production cost is revealed to be sufficiently low. This feature is modelled by assuming that the unit production cost is a random variable whose value becomes common knowledge *after* the monopolist has spent money on R & D and determined the expected profit-maximizing pricing policy. The fact that the monopolist has already spent a non-recoverable amount on R & D makes him worry about the outcome of his investment. In other words, if the value of the unit production cost turns out to be too high for production to be profitable, the monopolist will not be able to cover his sunk cost simply because production will not take place. It is reasonable to argue, therefore, that the monopolist would like to "insure" his assets against this possibility.

The monopolist can achieve this aim if there is a distribution of consumer types indexed by their valuation of the good with each consumer type experiencing a reduction in his utility by purchasing and consuming the good later than others. Then, the higher the type of a consumer (the higher his product valuation), the more this type's utility will be reduced by purchasing and consuming the good later than others. Plausible stories may include the following [Ireland and Stoneman (1985)]:

If the buyers are firms in a single industry and the product a new technology, a reason for wishing to buy early (before others) would be in order to gain a dominant market position in that industry. Also early buyers may be able to select the best locations in which to use the new technology, or indeed be able to select a range of products to be produced with the new technology with an unimpeded choice of locations in terms of the space of product characteristics. Furthermore, early buyers may be able to acquire complementary factors of production, such as labour with specific skills, more easily or cheaply than later buyers. Alternatively, the problem could involve a Veblen good, perhaps a consumer good such as holiday to a newly developing resort when early consumers obtain prestige but later ones do not. [Ireland and Stoneman (1985, p. 8)].

Assuming that the monopolist cannot identify consumers by their valuation, the monopolist will try to exploit the consumers' time preference by charging consumer types with a sufficiently high valuation for the product a non-refundable prepayment payable before the unit production cost becomes common knowledge and, at the same time promising them to the first to get the good if it is produced, in which case, they will also pay a unit price (payable upon delivery). For these types, the expected benefits from getting the good sooner than others outweigh the risk of losing the prepayment and not getting the good in the case where the unit cost turns out to be too high for production to be profitable. On the other hand, those consumer types for whom the expected benefits from early delivery do not outweigh the risk of losing the prepayment will choose not to pay the prepayment and, consequently, they will be able to buy the good (if it is produced) only after those who have prepaid are served.

An important implication of the above model (which will be discussed in more detail in the conclusion of the thesis where we will consider possible extensions of the model) is that the prepayment constitutes a mechanism by which the producer can shift the risk involved in the activity of innovation to the consumer. This is important in the face of arguments that have been made concerning the bias of the market against risky R & D.

The remaining three chapters of the core of the thesis examine the effects of prepayments in an environment where a monopolistic firm tries to protect its monopoly power against potential entry. Marriott's advance-purchase discount strategy is an application of the theory developed in chapter 4. Chapter 4 examines the role of prepayments as an integral part of the pricing strategy of an incumbent firm who is privately informed as to the level of cost and is concerned with deterring entry in a market of a new product. The need for a pricing policy that will be effective in deterring entry emanates from our assumption that the incumbent is

privately informed as to the level of cost. Thus, the entrant will consider entering the market only in the event that his cost is lower than the incumbent's cost.

Our interest will be focused on the game between the incumbent and the consumers rather than the entrant. The entrant will make his entry decision only after this game has reached an end and he will do so "passively". By this we mean that if consumers decide to purchase from the incumbent then the entrant will not enter since there will be no market left. If, on the other hand, consumers do not purchase from the incumbent then the entrant will enter to serve the entire market. The incumbent, therefore, wants to convince consumers to purchase the product from him so that he can eliminate the threat of entry.

Given the presence of cost uncertainty, consumers will not buy from the incumbent unless they know that his cost of production is lower than that of the entrant. Suppose then that, before production starts and before the entrant makes his entry decision, the incumbent is prepared to sign a number of forward contracts requiring buyers who sign to prepurchase one unit of the product by *prepaying*. The number of contracts to be signed is chosen by the incumbent and it can be less than the total number of consumers. Furthermore, contracts are signed on a first-come-first-served basis. Any consumers who do not have the chance to prepay will buy the product later by paying the monopoly price (which will be equal to the consumers' reservation price in the case of identical consumers).

After consumers get to know this pricing policy, they try to make an inference about the incumbent's true cost based on the size of the prepayment and the number of consumers who are allowed to prepay. The low-cost incumbent then will choose these variables in a way that would be unprofitable if the incumbent had high costs. In the separating equilibrium, therefore, consumers will buy from the low-cost incumbent only. Some of them will have the chance to prepay for one unit of the product. The entrant then will not enter so that the

remaining consumers will make their purchases of the product after it is produced by paying a monopoly price. The fact that some consumers prepay is taken by the entrant to mean that the incumbent's type is a low-cost one. Then the existence of some consumers who do not have the chance to prepay but, instead, wait until the end of the production period in order to purchase the product does not prompt the entrant to enter. Nevertheless, the entrant could still enter since, by entering, he would, at worst, be as well off as by staying out. To rule out a situation like the above, where the entrant enters even though he knows that he enters against a more efficient producer, we will assume that there is a very small fixed cost of entry.

Chapter 5 extends the previous model by assuming that consumers are heterogeneous and indexed by their reservation price. It is further assumed that the incumbent has two choices as to the pattern of serving consumers who are willing to prepay: the *efficient-serving* scheme and the *proportional-serving* scheme. Under the former, only consumers with the highest reservation prices are allowed to prepay while in the case of the latter, all consumers have the same probability of prepaying. Depending on the type of the serving scheme that is chosen by the incumbent, *rationing* of consumers in the equilibrium can be optimal from the point of view of the low-cost incumbent: only a part of the demand for advance-purchase contracts involving prepayments is satisfied by the low-cost incumbent in the separating equilibrium. If rationing actually takes place in the separating equilibrium, rationed consumers will be able to purchase the product from the low-cost incumbent after production has taken place by paying a monopoly price which will be higher than the prepayment.

Rationing will turn out to be optimal only in the case of the proportional-serving scheme where the low-cost incumbent will prefer to serve only a subset of consumers who are willing to prepay. Furthermore, the assumption of heterogeneous buyers and the availability of two

serving patterns to choose from will give rise to a *unique* separating equilibrium outcome as opposed to chapter 4 where there is a multiplicity problem even after the elimination of dominated strategies.

In chapter 6, it is the entrant's cost (rather than the incumbent's) which is the unknown parameter. We will show that an incumbent seller with a known unit cost can reduce the probability of entry into his market of an entrant who faces a sufficiently large fixed cost of entry and whose variable unit cost is unknown by signing contracts requiring buyers to pay for the product before it is delivered to them. This model assumes that there are a single incumbent, a single potential entrant and two buyers. The incumbent and the buyers do not know the entrant's cost which may turn out to be either higher or lower than the incumbent's unit cost. In order to forestall a situation where the entrant's cost will turn out to be lower than the incumbent's (with the possible consequence of provoking entry), the incumbent will try to "appropriate" either one or both buyers before the time of entry decision thus reducing the probability of entry since the presence of fixed costs of entry implies that the entrant's average cost is decreasing in the number of customers served. If the incumbent is successful in convincing the buyers to purchase the product from him then it might be the case that the incumbent's pricing policy will facilitate the entry prevention of a *more* efficient producer.

The incumbent's pricing strategy requires buyers to prepay for their purchases of the product before the start of production (and thus before the entrant learns his cost and makes his entry decision). Why would buyers ever consider prepaying instead of waiting for the entrant's decision? The answer to this question hinges on two critical points. First, we will assume that each buyer's reservation price depends on the number of buyers that purchase the product. Each buyer's reservation price for the product is greater when only that buyer purchases the product than when both buyers purchase the product. Hence, each buyer has

the incentive to prepay (hoping that the other buyer will not prepay) since if he does not prepay and wait for the entrant's decision then the entrant may enter in which case the product will be sold to both buyers. Second, given each buyer's incentive to prepay because of the above reason, each buyer goes through the following reasoning:

If I do not prepay and the other buyer prepays then the probability of entry will be reduced in which case I may have to either pay a higher price in order to get the product from the incumbent in a later period or do without the product if the incumbent's monopoly price in that period is greater than my reservation price.

It will be shown that, depending on the values of the parameters of the model, one of the following possibilities can emerge as a subgame-perfect equilibrium:

- (1) A single buyer prepays
- (2) Both buyers prepay
- (3) Neither buyer prepays

If a single buyer prepays, the probability of entry is reduced while if both prepay, entry is completely eliminated.

We conclude this section by pointing out that much of this thesis is concerned with the issue of explaining the reasons why prepayments are part of the pricing policies of monopolistic firms operating in a variety of environments. Nevertheless, we will also address the issue of the welfare effects of prepayments. At this point, we will limit ourselves to the remark that prepayments can be either welfare-increasing or welfare-decreasing depending on the environment in which they are used.

PART 2

ANALYSIS OF PREPAYMENTS

2 CONTRACTUAL EXCHANGE OR VERTICAL INTEGRATION? THE ROLE OF PREPAYMENTS IN SUPPORTING EFFICIENT CONTRACTUAL EXCHANGE

1. Introduction

This chapter's theme is the analysis of an example of a *market* exchange between a buyer and a seller under conditions of uncertainty and asset specificity. As this type of exchange will yield an 'inefficient' outcome, two alternatives will be examined: *contractual* exchange and *vertical integration*. Contractual exchange will play an important role since contracts stipulating *prepayments* will support efficient exchange in some situations.

The product in question is produced by a special-purpose technology which requires an investment in transaction-specific durable assets. Asset specificity refers to durable investments that are undertaken in support of particular transactions. The opportunity cost of such investments is much lower in best alternative uses (or by alternative users) should the original transaction be prematurely terminated [Williamson (1985, p. 95)]. Furthermore, the specific identity of the parties to a transaction matters in these circumstances, which is to say that continuity of the relationship is valued [Williamson (1985, p. 55)].

Transaction-specific investments create a problem for market procurement. An investment in assets specific to a transaction locks both the buyer and the seller into a bilateral monopoly because the cost of the asset cannot be fully recovered if the transaction is terminated. In the

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terminology of Klein, Crawford and Alchian (1978, p. 298), a stream of quasi-rents is created which can be appropriated through opportunistic behaviour by either party.

The second attribute of the model is the presence of uncertainty in the demand side of the exchange. There are two periods in the model. The buyer places an order in period 1 and the producer makes the specific-asset investment. Production occurs in period 2. The buyer, however, can either take delivery, paying the unit price of the product, or refuse it. This behaviour hinges on the assumption that demand is stochastic. The gross value of the good will be assumed to be uniformly distributed over the interval $[0,1]$ and the two parties to the exchange learn this value in the beginning of period 2. If the realized value of the product is such that the buyer prefers to cancel the order and defect from the agreement under the specific-asset technology, the producer will be holding unproductive assets which cannot be economically shifted to other uses. The prospect of this loss will prompt the producer to make efficiency-distorting price adjustments.

The influence of the previously mentioned kind of uncertainty on economic organization is conditional. The existence of uncertainty is a matter of little consequence for non-specific transactions. Since the seller can easily redeploy his assets through new trading relations, continuity of a particular relationship has little value. Similarly, the presence of asset specificity in an environment without uncertainty does not cause any problems since the supplier will not commit any specific assets unless the gross value of the good is greater than or equal to the marginal cost of its production.

As a result, whenever assets are specific in a non-trivial degree, increasing the degree of uncertainty makes it more imperative to organize transactions within governance structures that have the capacity to "work things out" [Williamson (1985, p. 79)]. We will examine two such governance structures: *contractual* versus *internal* exchange. We will assess the

effectiveness of both modes of organization in dealing with the inefficiencies caused by the lack of *safeguards* in the *market* exchange between the buyer and the seller.

Since internal organization of this exchange through vertical integration yields the efficient outcome, vertical integration is going to be the benchmark with which different contracting alternatives will be compared. These contracting schemes will share the characteristic of explicitly introducing safeguards against the buyer's potential opportunistic behaviour.

The particular safeguard that we are interested in examining is *prepayments*: the buyer is charged an up-front payment when he places the order. Then we want to assess the role of contracts stipulating prepayments in supporting efficient exchange.

1.1. Contractual Exchange: Some Introductory Remarks

The contracts that we are concerned with are *complete* in that they specify each party's obligations in every conceivable eventuality. The benefits of writing such contracts derive from the fact that they lay down each party's obligations completely and unambiguously in advance.

There are costs associated with writing complete contracts, however. The following seem to be important [Hart and Holmstrom (1987)]: (1) the cost of deciding and reaching an agreement about how to deal with all possible events; and (2) the cost of writing the contract in a sufficiently clear and unambiguous way so that the terms of the contract can be enforced. Less important seems to be the cost to each party of waiting for the various eventualities to occur - i.e., the realization of the value of the good in the beginning of period 2 - as long as the length of the periods is not too long.

As a consequence of completeness, the contracts that we will examine are also *self-*

enforcing [Hart and Holmström (1987)] and of the *bilateral-governance* type [Williamson (1985)]. Both concepts refer to those contracts that are not enforced by outsiders or third parties, such as the courts. Although the courts may be there as a last resort, these agreements are enforced on a day-to-day basis by custom, good faith, reputation, and so on. In our case, it is the completeness of contracts that precludes the need to resort to third-party assistance.

The consideration of such contracts as an alternative to the market exchange in question is motivated by the fact that this market exchange will give rise to an inefficiency: the product will be exchanged at a price that exceeds marginal cost. In other words, there are going to be some realizations of the value of the good above marginal cost for which efficient exchange will not take place since the unit price of the product will exceed those values.

As a remedy to this inefficiency, we will consider contract φ below as well as some variants of it:

(φ) The buyer places an order in period 1 and pays a prepayment which is part of the full payment. The producer makes the specific-asset investment in period 1 and receives the prepayment which is non-refundable. If the buyer confirms the order in period 2, the producer also receives the remaining part of the full payment.

The introduction of the prepayment as a safeguard device will support an efficient outcome. The buyer will confirm the order and exchange will take place whenever the realized value of the product is greater than or equal to the marginal cost of production. This will be true as long as the prepayment is set equal to the amount of the investment in specific assets.

1.2. Contractual Exchange when the Seller Values the Prepayment Less than the Buyer

Contracting scheme ρ illustrates the importance of a complete contract when there are relationship-specific investments. There are situations, however, where it is difficult to postulate contracts that induce efficient relationship-specific investments and this difficulty may become an important factor in explaining vertical integration.

Our type of contracts will give rise to inefficiencies in situations where the seller values the prepayment less than the buyer: even though the buyer pays a prepayment which is equal to the investment in specific assets (this will be referred to as the *full* value of the prepayment), the value of this prepayment to the seller is less than the amount of the investment. Failure to assure the seller that a prepayment which he values at a full amount will be transferred to him will prompt the seller to safeguard his specific-asset investment through efficiency-distorting price adjustments.

We will consider four variations of contract ρ depending on (1) whether the full value of the prepayment is reduced by an amount proportional to that value or by a constant amount and (2) whether the full value of the prepayment is reduced under all possible demand realizations or under those realizations that induce the buyer to cancel the order.

It is going to be true that the efficiency properties of contract ρ will carry over to the case where the seller values the prepayment less than the buyer only when the prepayment is reduced by a constant amount and regardless of whether the buyer confirms the order or not. The remaining three contracts will give rise to inefficient outcomes. Depending on the contract considered, it may be the case that either inefficient exchange takes place or efficient exchange does not take place.

We conclude therefore that there are situations where contracts cannot support efficient exchange. Internal organization of the transaction through vertical integration becomes then a more appropriate mode of economic organization.

Before proceeding with the formal analysis, in section 2 we will present some theoretical and empirical evidence that is consistent with the type of contracts considered in point (2) above. Section 3 describes the main ingredients of the model while the market exchange in question is analyzed in section 4. In section 5 we describe the internal organization (through vertical integration) of the market exchange. Contracting scheme ρ is examined in section 6 and its variants (when the seller values the prepayment less than the buyer) in section 7. Section 8 compares our findings with those of Williamson (1983) and section 9 presents some concluding remarks.

2. Reputation Effects and Contract-Handling Agents: Why the Seller May Value the Prepayment Less than the Buyer

Two of the four variations of contract ρ that will be considered whenever the seller values the prepayment less than the buyer refer to cases where either the full value of the prepayment to the seller is reduced only when the order is cancelled or regardless of whether or not the order is cancelled.

One way of interpreting the seller's reduced valuation of the prepayment whenever the order is cancelled is as reflecting the potential loss of valuable opportunities for future trade if the cancellation of the present partnership becomes a factor that associates the seller with a reputation of being an unreliable trader. Furthermore, if we assume that assets are of the "semi-specific" type [Williamson (1985, p. 54)], the reduced value of the prepayment may also reflect the search and transaction costs of finding a new trading partner and thus redeploying the assets that were left unproductive due to the termination of the relationship with the previous transactor.

On the other hand, the reduced prepayment valuation which occurs under all possible demand realizations can be thought of as reflecting the cost of third-party consultation on how to formulate and write the contract in question.

Reputations have been considered in the economics and legal literature to be private devices or non-legal sanctions which provide incentives that assure contractual performance in the absence of third-party enforcer [f.ex., Klein and Leffler (1981, p. 616), Cherny (1990, p. 393) and Williamson (1991, p. 290, 291)].² Furthermore, in order for a reputation to have an effect, both sides involved in a transaction must *ex ante* have some idea of the meaning of appropriate fulfilment of the contract. Potential future trading partners must be able to observe either fulfilment or lack of it [Kreps, in Alt and Shepsle eds. (1990, p. 93)].

The issue of reputation-activated contract enforceability is not relevant to our context since the confirmation of an order depends mainly on a random event. Rather, order cancellation or non-enforceability bring into existence reputation effects. Nevertheless, if the seller is at all likely to be discredited as not being a decent and reliable trader, we must make the assumption that outsiders to the contract (e.g., other firms in the industry) do not have and cannot obtain perfect information as to the precise reason of the cancellation of the order.

Turning to the case where the seller values the prepayment less than the buyer regardless of whether the order is cancelled or not, there are several examples which support our position that the reduction in the full value of the prepayment may reflect payments to third parties responsible for formulating the contract. One such example concerns the American Information Exchange (AMIX) [Orr (1992)]. AMIX is an online information market used by customers to find products and services. Customers can also put their requirements and take bids. What is relevant to our case is that AMIX handles contracts and payments taking a cut of the value of each deal.³

3. The Model⁴

3.1. Technologies and Costs

Let us assume that the product in question is produced by a special-purpose technology (T) which requires an investment in transaction-specific durable assets. Asset specificity implies that there is going to be a non-salvageable value in any advance commitments that the producer makes. This value will be denoted by k . Furthermore, the value that can be realized by redeploying the salvageable part of the investment will be given by v . Alternatively, the redeployable unit operating costs of T are v .⁵

3.2. Demand and the Contracting Process

There are two periods. Orders are placed in the first, and production, if any, occurs in the second. Demand is stochastic in that the gross value of the good to the buyer (r) is assumed to be uniformly distributed over the interval $[0,1]$. The quantity demanded at every price will be assumed to be a constant, which it will be convenient to set equal to unity. The product value and cost relations are shown in figure 2.

The actual value of r is realized in the beginning of period 2. That is, after the buyer has placed the order and the seller has committed the investment in specific assets. Depending on the realized value and the price of the product, the buyer can either take delivery or refuse it. In the latter case, the seller will be left with unproductive assets.

The market exchange under discussion can be described as follows.

(m) The buyer places an order in period 1 and the producer makes the specific-asset investment. If the buyer confirms the order, the producer receives a per unit payment of p^m in the second period but nothing otherwise.⁶

The next section analyzes the consequences of organizing the transaction in question within the market. We will assume that there is competition in the supply side of the market and a single buyer. This is an *ex ante* consideration since the condition of asset specificity implies that a bilateral monopoly is created after a transaction has been initiated between the buyer and a particular seller.

4. Market Exchange m

When the transaction is organized in the market, the buyer will confirm the order whenever the realized value of the good is greater than or equal to p but not otherwise. The buyer's expected benefits then are equal to

$$\begin{aligned} E\pi_B &= \int_p^1 (r - p) dr \\ &= \frac{1}{2} (1 - p)^2. \end{aligned} \quad (1)$$

Competition in the supply side of the market and a single buyer imply that the buyer's equilibrium expected benefits should be positive. In contrast, if we were to assume that the market consists of many buyers then the equilibrium expected benefits for each buyer would be zero.

The supplier's expected profits are equal to

$$\begin{aligned} E\pi_s &= - \int_0^p k dr + \int_p^1 (p - v - k) dr \\ &= -kp + (1-p)(p - v - k) \\ &= (1-p)(p - v) - k. \end{aligned}$$

Since the production side of the industry is competitively organized, producers will be willing to supply if

$$(1-p)(p-v) - k = 0$$

and hence

$$p^m = v + \frac{k}{1-p^m}. \quad (2)$$

For any p^m in $(0,1)$ we have that $p^m > v$. The product thus will be exchanged at a price that exceeds marginal cost. In other words, for any realized value of the good in $[v, p^m)$, exchange does not take place although such exchange would be efficient according to the marginal-cost supply criterion.

The obtained result is due to the lack of any device that could safeguard the seller's specific-asset investment against the possibility of the buyer reneging on his order. This prompts the seller to raise the unit price of the product in order to cover both variable and fixed costs. Indeed, it can be seen from (2) that $p^m > v + k$.

The increase in the unit price, on the other hand, raises the probability that the buyer will cancel his order. This is of no concern to the seller, however, since, by assumption, the seller

breaks even in expected terms. The disadvantage, therefore, accrues entirely to the buyer who derives a lower expected benefit in equilibrium [as can be seen from (1)].

In an effort to restore efficiency in our two-party transaction, we will consider two alternative governance structures in the next two sections: vertical integration and contracting scheme β for the case where the seller values the prepayment in the same amount as the buyer.

5. Vertical Integration

The integrated firm will decide to produce only if the realized value of the product is greater than or equal to the marginal cost, v . Hence, the probability of production under T is $1 - v$. The average net benefits during production periods are $(1 - v)/2$. In computing expected net benefits, the amount of the investment in specific assets, k , must be taken into account. Thus, expected net benefits for technology T are

$$\begin{aligned} b_T &= (1 - v) \frac{1 - v}{2} - k \\ &= \frac{(1 - v)^2}{2} - k \end{aligned}$$

Contracting scheme β which is analyzed below will replicate the vertical integration conditions, being thus in accordance with the marginal-cost supply criterion and generating an expected net benefit for the buyer equal to b_T .

6. Contracting Scheme β

Let us restate contract ρ :

- (ρ) The buyer places an order in period 1 and pays a prepayment equal to p_1^{ρ} . The producer makes the specific-asset investment in period 1 and receives the prepayment which is non-refundable and is part of the full payment for one unit of the product, p^{ρ} . If the buyer confirms the order in the beginning of period 2, the producer also receives the remaining part of the full payment, $p^{\rho} - p_1^{\rho}$.

Let $p - p_1$ be denoted by p_2 ; p_2 is received in the second period and only if the buyer confirms the order. The buyer will confirm the order under contract ρ whenever the realized value of the good is greater than or equal to p_2 . Then, the buyer's expected net benefits are

$$\begin{aligned} E\pi_B &= - \int_0^{p_2} p_1 dr + \int_{p_2}^1 (r - p_1 - p_2) dr \\ &= -p + \frac{1}{2}(1 + p_2^2). \end{aligned}$$

Let u_B^{ρ} denote the equilibrium expected net benefits of the buyer:

$$u_B^{\rho} = -p + \frac{1}{2}[1 + (p_2^{\rho})^2]. \quad (3)$$

Given that $p = p_1 + p_2$ and, both p_1 and p_2 take values in $(0,1)$, the equality in (3) implies that $u_B^{\rho} \in (0,0.5)$.

As has been explained in section 4 for the case of market exchange, the existence of a competitively organized production market and a single buyer implies that u_B^{**} should be

positive.

Given that the seller receives p_1 and bears the cost of the investment in specific assets (k) regardless of whether the buyer confirms the order or not while he receives an additional p_2 and incurs the marginal cost v when the buyer confirms the order, his expected profits are equal to the following:

$$\begin{aligned} E\pi_s &= p_1 - k + \int_{p_2}^1 (p_2 - v) dr \\ &= p - p_2 - k + (1 - p_2)(p_2 - v) \end{aligned} \quad (4)$$

$$\begin{aligned} &= -u_B + \frac{1}{2}(1 + p_2^2) - p_2 - k + (1 - p_2)(p_2 - v) \\ &= -\frac{1}{2}p_2^2 + vp_2 - (v + k) + \frac{1}{2} - u_B \end{aligned} \quad (5)$$

using (3) and the definition of p_2 , (5) implies that the seller also incurs the cost of guaranteeing to the buyer a net benefit of u_B .

One way to solve the model is to set the expression in (4) equal to zero since, by assumption, competition prevails among suppliers of the product. This procedure would not be helpful, however, since it would only yield a set of (p, p_2) pairs but not a unique solution and thus not a unique contracting scheme of type ρ . One could argue that the multiplicity of equilibrium solution pairs does not create any problems as long as the vertical-integration solution belongs to that set. Then we could just pick that solution from the set of (p, p_2) pairs on the grounds that it is the desirable one.

We argue, however, that this solution procedure does not provide sufficient justification and

support to the argument that contracting scheme p is an efficiency-promoting governance structure.

In order to be able to obtain a unique solution and, hence, a unique contractual scheme (which supports efficient exchange) in an optimal way so that we do not have to just choose from among a set of equilibrium solutions, we will interpret the assumption of the competitively organized production side of the industry in a more constructive way.

To this end, we argue that it is the *full payment* (p) that should be consistent with zero expected profits. The supplier should not be prevented from optimally choosing either component (p_1 or p_2) of the full payment as long as this full payment yields zero equilibrium expected profits.

We will follow this alternative solution procedure starting with the maximization of expected profits [as they are given by (5)] with respect to p_2 :

$$\max_{p_2} E\pi_s$$

The first-order condition for a maximum yields

$$-p_2 + v = 0$$

from which we have that

$$p_2^p = v. \quad (6)$$

Substituting back into (5) and using (3) we get an expression for the expected profits as a function of the full payment, p :

$$\begin{aligned}
 E\pi_3(p; v, k) &= -\frac{1}{2}v^2 + v^2 - (v + k) + \frac{1}{2} \cdot p - \frac{1}{2}(1 + v^2) \\
 &= p - v - k.
 \end{aligned}
 \tag{7}$$

Now we set this expression equal to zero since we have assumed that the supply side of the market is competitively organized:

$$p^0 = v + k. \tag{8}$$

Since $p_2^0 = p^0 - p_1^0$, we have that $p_1^0 = k$. Furthermore, from (3), the equilibrium expected net benefits of the buyer are

$$\begin{aligned}
 u_B^0 &= -v - k + \frac{1}{2}(1 + v^2) \\
 &= \frac{(1 - v)^2}{2} - k
 \end{aligned}$$

which is identical to the net benefit calculation, $b_{T,v}$ for technology T under the vertical integration reference structure.

Since the buyer confirms the order whenever the realized value of the good is greater than or equal to p_2^0 and since $p_2^0 = v$, we have that contract ρ supports efficient exchange in that orders will be confirmed whenever the realized value of the good is greater than or equal to marginal cost (v). In other words, contract ρ , stipulating a prepayment which is equal to the investment in specific assets ($p_1^0 = k$), replicates the efficient supply condition of vertical integration. Furthermore, our solution procedure guarantees that contract ρ is the unique contract that supports efficient exchange between the two parties in the transaction under

consideration.

Our result is in agreement with intuition. Since the producer's investment in specific assets is completely safeguarded against the possibility of order cancellation and the producer operates in a competitively organized industry, we would expect that the full payment (p^p) should be enough to cover variable and fixed costs.

The following proposition summarizes the main result of this section.

Proposition 1. *For any v , k and u_B^p such that $v \in (0,1)$, $k \in (0,1)$, $u_B^p \in (0,0.5)$ and $E\pi_v(p,v,k) \geq 0$, there exists a unique contract of type φ which, accompanied by the stipulation that $p_1^p = k$, replicates the efficient-supply condition of vertical integration.*

Taking into consideration that contract φ is comprehensive and easy to write as well as that internal organization is characterized by incentive and bureaucratic disabilities, contract φ should be the most preferable mode of organization of the two-party transaction in question.

7. Contracting Scheme φ When the Supplier Values the Prepayment Less than the Buyer

In this section, we extend our model to the case where the seller values the prepayment less than the buyer. In section 2, we described examples of transactions that are consistent with the seller's reduced valuation of the prepayment. Specifically, the reduced valuation may reflect either the loss of the seller's reputation for being a reliable trader or payments to third parties responsible for the formulation and writing of the contract.

In order to assess whether or not prepayments can support efficient exchange in situations like the above, we should adapt contract \wp to the needs of the analysis of this section. Hence, contract \wp will be subject to either of the following four variations depending on (i) whether the full value of the prepayment is reduced by an amount proportional to that value or by a constant amount, and (ii) whether the full value of the prepayment is reduced under all possible demand realizations or under those realizations that imply cancellation of the order:

($\wp 1$) The full payment for one unit of the product is p^{w1} . The buyer places an order in period 1 and prepays p_1^{w1} . The producer makes the specific-asset investment in period 1 and receives p_1^{w1} if the buyer confirms the order. He values the received prepayment in amount αp_1^{w1} , $0 < \alpha < 1$, if the buyer cancels the order. If the buyer confirms the order, the seller also receives an additional period-2 payment equal to p_2^{w1} such that $p_1^{w1} + p_2^{w1} = p^{w1}$.

($\wp 2$) The full payment for one unit of the product is p^{w2} . The buyer places an order in period 1 and prepays p_1^{w2} . The producer makes the specific-asset investment in period 1 and receives p_1^{w2} if the buyer confirms the order. He values the received prepayment in amount $p_1^{w2} - t^{w2}$, $t^{w2} \in (0, p_1^{w2})$, if the order is cancelled. If the buyer confirms the order, the seller receives an additional period-2 payment equal to p_2^{w2} such that $p_1^{w2} + p_2^{w2} = p^{w2}$.

($\wp 3$) The full payment for one unit of the product is p^{w3} . The buyer places an order in period 1 and prepays p_1^{w3} . The producer makes the specific-asset investment in period 1 and values the received prepayment in amount αp_1^{w3} , $0 < \alpha < 1$, regardless

of whether or not the buyer confirms the order. If the order is confirmed, the seller is also paid $p_2^{w^j}$ in period 2, where $p_1^{w^j} + p_2^{w^j} = p^{w^j}$.

- ($\wp 4$) The full payment for one unit of the product is p^{w^d} . The buyer places an order in period 1 and prepays $p_1^{w^d}$. The producer makes the specific-asset investment in period 1 and values the received prepayment in amount $p_1^{w^d} - t^{w^d}$, $t^{w^d} \in (0, p_1^{w^d})$, regardless of whether or not the buyer confirms the order. If the order is confirmed, the seller is also paid $p_2^{w^d}$ in period 2, where $p_1^{w^d} + p_2^{w^d} = p^{w^d}$.

Contracts $\wp 1$ and $\wp 2$ imply that the seller values the prepayment less than the buyer whenever the buyer cancels the order. These contracts, therefore, reflect the adverse reputation effects coming from the cancellation of orders. In contrast, contracts $\wp 3$ and $\wp 4$ reflect third-party payments since the seller values the prepayment less than the buyer regardless of whether or not the order is cancelled.

7.1. Contracting Scheme $\wp 1$

We start the analysis of this contract with the buyer's expected benefits from placing an order. The calculation of these benefits, however, is not affected by the new considerations since it is only the seller who values the prepayment at an amount which is less than its face value. Hence, the buyer's expected net benefits in equilibrium are given by (3) where \wp is substituted by $\wp i$, $i = 1, 2, 3, 4$. We proceed, therefore, with the seller's expected profits. These are given by⁷

$$\begin{aligned}
 E\pi_s &= \int_0^{p_1} (\alpha p_1 - k) dr + \int_{p_1}^1 (p_1 + p_2 - v - k) dr \\
 &= \frac{1}{2}(\alpha - 1)p_1^2 + \frac{1}{2}(1 - 2\alpha)p_1^2 + \frac{1}{2}[(\alpha - 1)(1 - 2u_B) + 2v]p_1 + \frac{1}{2} - u_B - v - k. \quad (9)
 \end{aligned}$$

Maximization of $E\pi_s$ with respect to p_1 yields the following first-order condition:

$$\frac{3}{2}(\alpha - 1)p_1^2 + (1 - 2\alpha)p_1 + \frac{1}{2}[(\alpha - 1)(1 - 2u_B) + 2v] = 0.$$

There are going to be two solutions for p_1 . Only one is acceptable, however, in that it is consistent with the interesting case where $p_1 \in (0, 1)$.⁸ This solution is given by the following equation:

$$p_1 = \frac{2\alpha - 1 - \sqrt{(1 - 2\alpha)^2 - 3(\alpha - 1)[(\alpha - 1)(1 - 2u_B) + 2v]}}{3(\alpha - 1)}. \quad (10)$$

By substituting for p_1 in (9) we get an expression for expected profits as a function of u_B (given the values of v , k , and α). Setting this expression equal to zero yields a solution for the buyer's equilibrium expected benefits $u_B^{B^I}$. Then, using this solution and equation (10) we can solve for $p_1^{B^I}$. A value for the full payment (p^{B^I}) can be then obtained using (3). Finally, the prepayment will be equal to $p^{B^I} - p_1^{B^I}$.

Table 1 shows the results of some numerical calculations that were performed following the procedure that has been described. The values of the parameters v and k have been chosen according to the restrictions that these parameters should obey in order for $p_1^{B^I}$ to be an

acceptable solution.

For the values of α that have been considered, our results are characterized by the following:

$$\begin{aligned}p_1^{p1} &> k = 0.15 \\p_2^{p1} &> v = 0.33 \\p^{p1} &> v + k = 0.48.\end{aligned}$$

Furthermore, as α increases, the values of p_1^{p1} , p_2^{p1} and p^{p1} converge from above to k , v and $v + k$, respectively. These are the values that were obtained under contract φ .

The resulted numbers are in agreement with intuition. The lower the value of α (or, in reputation terms, the higher the foregone opportunities for future trading are valued by the seller), the higher the full payment and its components are and the lower the equilibrium expected utility of the buyer is. In other words, if future potential trading partners believe (with a positive probability) that the order has been cancelled because of misconduct from the part of the seller, the seller will make up for the loss of foregone future trading by adjusting upwards the prepayment and the full payment that are stipulated in the contracts with his present trading partners.

These price adjustments result in an inefficiency: $p_1^{p1} > v$. The buyer will not confirm an order for any realized value of the good in $[v, p_2^{p1})$. For these values, therefore, efficient exchange does not take place.

More generally, in order to determine whether or not contract φ 1 supports efficient

exchange, we evaluate the first-order condition (FOC) at $p_2 = v$:

$$\begin{aligned} FOC_{p_2, v} &= \frac{3}{2}(\alpha - 1)v^2 + (1 - 2\alpha)v + \frac{1}{2}[(\alpha - 1)(1 - 2u_B) + 2v] \\ &= \frac{\alpha - 1}{2}[(3v^2 - 4v + 1) - 2u_B] \\ &= \frac{\alpha - 1}{2}\left[3(v - 1)\left(v - \frac{1}{3}\right) - 2u_B\right]. \end{aligned}$$

For any $v \in [1/3, 1]$, it is true that the FOC evaluated at $p_2 = v$ is positive. This implies that $p_2^{BI} > v$. Then, since the buyer confirms the order whenever the realized value of the good is greater than or equal to p_2^{BI} and since $p_2^{BI} > v$, it may be the case that efficient exchange will not take place. In other words, for any realized value in (v, p_2^{BI}) , the buyer will cancel his order although exchange between the parties would be efficient according to the marginal-cost supply criterion.

If $v \in (0, 1/3)$ then there are values of u_B in $(0, 0.5)$ for which the FOC evaluated at $p_2 = v$ is less than or equal to zero. If v actually takes such a value and the buyer's equilibrium expected utility is less than or equal to $(3/2)(v - 1)(v - 1/3)$ then we will have that $p_2^{BI} \leq v$. In other words, if the average cost of production can be kept to low levels and the buyer does not require too high an expected utility in equilibrium then the seller reduces p_2 in order to increase the probability of order confirmation. p_2 may be reduced too much, however, in that the buyer will confirm an order even under demand realizations in which exchange between the two parties is not efficient.

The previous discussion is summarized in proposition 2.

Proposition 2. Suppose that $0 < (\alpha - 1)(1 - 2u_B) + 2v < 1 + \alpha$, for some α , v and u_B such that $\alpha \in (0, 1)$, $v \in (0, 1)$ and $u_B \in (0, 0.5)$. Suppose further that k is such that $k \in (0, 1)$ and

$E\pi_3(u_0, \alpha, v, k) \geq 0$. Then, there exists a unique contract of type $\varphi 1$ where

- (i) if $v \in [1/3, 1]$ then $p_2^{w1} > v$; and
- (ii) if $v \in (0, 1/3)$ and $3(v - 1/3) - 2u_0 <, =, \text{ or } > 0$ then $p_2^{w1} <, =, \text{ or } > v$, respectively.

It is therefore possible that contract $\varphi 1$ can support efficient exchange although it seems that for most values of marginal cost, contract $\varphi 1$ may result to an inefficient outcome in that efficient exchange may not take place.

7.2. Contracting Scheme $\varphi 2$

Recall that, under this contract, the supplier receives p_1^{w2} in period 1 and p_2^{w2} in period 2 if the buyer confirms the order while if the buyer cancels the order, the supplier receives a prepayment which is valued by him in amount $p_1^{w2} - t^{w2}$.

The seller's expected profits are thus given by⁹

$$\begin{aligned} E\pi_3 &= \int_0^{p_1} (p_1 - t^{w2} - k) dr + \int_{p_1}^1 (p_1 + p_2 - v - k) dr \\ &= -\frac{1}{2}p_2^2 + (v - t^{w2})p_2 - (v + k) + \frac{1}{2} - u_0 \end{aligned} \quad (11)$$

Maximizing $E\pi_3$ with respect to p_2 gives the following first-order condition:

$$-p_2 + v - t^{w2} = 0$$

which implies a value for p_i equal to

$$p_i^{B2} = v - t^{B2}. \quad (12)$$

Substituting for p_i^{B2} into (11) and using (3), we get

$$\begin{aligned} E\pi_s(p_i, v, k, t^{B2}) &= -\frac{1}{2}(v - t^{B2})^2 + (v - t^{B2})^2 - (v + k) + \frac{1}{2} + p - \frac{1}{2}[1 + (v - t^{B2})^2] \\ &= p - (v + k). \end{aligned}$$

The assumption of competition in the supply side of the market implies that

$$E\pi_s(p_i, v, k) = 0$$

which gives

$$p^{B2} = v + k. \quad (13)$$

Given (12) and (13), the prepayment p_i^{B2} is equal to

$$p_i^{B2} = p^{B2} - p_i^{B2} = k + t^{B2}. \quad (14)$$

From (13) we can conclude that the fact that the seller receives a prepayment which is valued by him at $p_i^{B2} - t^{B2}$ in cancellation states does not alter the full payment that the seller receives in confirmation states. The full payment is still equal to the one that the supplier receives under contract p where $\alpha = 1$. This is intuitive since the value of the prepayment to the seller is not reduced by any amount whenever the buyer confirms the order.¹⁰

Although the full payment remains unchanged, the prepayment is raised by the constant t^{p2} above the amount of investment in specific assets. In this way the seller eliminates the risk of getting a prepayment reduced by t^{p2} if the buyer cancels the order.

Since the full payment remains unchanged and the prepayment increases by t^{p2} , the period-2 payment (p_2^{p2}) must be reduced by t^{p2} relative to its contract- φ value. Thus, contract $\varphi 2$ may give rise to an inefficiency which is different from the one that contract $\varphi 1$ gives rise to. Specifically, for any realized value of the good in (p_2^{p2}, v) , the buyer confirms the order and exchange takes place although the value of the good is lower than the marginal cost of production.

The following proposition summarizes the results for contract $\varphi 2$.

Proposition 3. *For any v, k, u_b^{p2} and t^{p2} such that $v \in (0, 1)$, $k \in (0, 1)$, $u_b^{p2} \in (0, 0.5)$, $t^{p2} \in (0, p_1^{p2})$ and $E\pi_S(p; v, k, t^{p2}) \geq 0$, there exists a unique contract of type $\varphi 2$ which postulates that*

$$\begin{aligned} p_1^{p2} &= k + t^{p2} \\ p_2^{p2} &= v - t^{p2} \\ p^{p2} &= v + k. \end{aligned}$$

7.3. Contracting Scheme $\varphi 3$

Under contract $\varphi 3$ (and that of $\varphi 4$), the supplier values the prepayment less than the buyer regardless of whether or not the buyer confirms the order. A relevant example refers to the case where the supplier has to pay a fee to third parties responsible for the legal formulation of the contract.

To analyze the implications of this contract, we follow the same procedure as with the previous contracts. Thus, the seller's expected profits are¹¹

$$E\pi_2 = \int_0^{p_2} (\alpha p_1 - k) dr + \int_{p_2}^1 (\alpha p_1 + p_2 - v - k) dr \\ = \left(\frac{\alpha}{2} - 1 \right) p_2^2 - (\alpha - 1 - v) p_2 + \frac{\alpha}{2} - \alpha u_B - (v + k). \quad (15)$$

Maximizing $E\pi_2$ with respect to p_2 gives the following first-order condition:

$$2 \left(\frac{\alpha}{2} - 1 \right) p_2 - (\alpha - 1 - v) = 0$$

which implies a value for p_2 equal to

$$p_2^{*1} = \frac{v + (1 - \alpha)}{2 - \alpha}. \quad (16)$$

Substituting for p_2^{*1} into (15) and using (3), we get $E\pi_2(p; v, k, \alpha)$. By setting this function equal to zero, we obtain a solution for p which is equal to the following:¹²

$$p^{*1} = \frac{(\alpha - 1)[v + (1 - \alpha)]^2}{\alpha(2 - \alpha)^2} + \frac{v + k}{\alpha}. \quad (17)$$

Given (16) and (17) the prepayment is equal to

$$p_1^{p3} = p^{p3} - p_2^{p3} \\ = \frac{v + (1 - \alpha)}{2 - \alpha} \left\{ \frac{(\alpha - 1)[v + (1 - \alpha)]}{\alpha(2 - \alpha)} - 1 \right\} + \frac{v + k}{\alpha}. \quad (18)$$

As can be seen from (16), contract $p3$ gives rise to the same sort of inefficiency as contract $p1$: $p_1^{p3} > v$, i.e., for any realized value to the buyer in $[v, p_1^{p3}]$, the order will be cancelled and exchange between the parties will not take place although such exchange would be efficient.

Although $p_2^{p3} > v$ for any $\alpha \in (0, 1)$, $v \in (0, 1)$ and $k \in (0, 1)$, the relation between the equilibrium full payment (p^{p3}) and total unit costs ($v + k$) as well as that between the equilibrium prepayment (p_1^{p3}) and the amount of investment in specific assets (k) depends on the values of v , k and α .

First, we explore the relation between the equilibrium full payment and total unit costs [using (17)] and that between the equilibrium prepayment and the investment in specific assets [using (18)], given a value for α in $(0, 1)$. Figure 3 summarizes the results of these comparisons.¹³

We can see that if k is sufficiently high (i.e., $k \geq k_2$) then we have that $p^{p3} > v + k$ and $p_1^{p3} > k$ regardless of the value of v . An analogous result applies when $v > v_2$: we have that $p^{p3} > v + k$ and $p_1^{p3} > k$ regardless of the value of k . Thus, if the marginal cost of production and the amount of investment in specific assets are sufficiently high, the seller hedges against the prospective loss of $(1 - \alpha)p_1$ by increasing both the full payment and the prepayment above their respective values of $v + k$ and k .

If k and v take intermediate values ($k_1 \leq k < k_2$ and $v_1 < v < v_2$), the prepayment becomes

smaller than the specific-asset investment. This effect reduces the full payment which, however, remains at a level above that of total costs. This is due to the fact that $p_j^{s,j} > v$.

Finally, if k and v become sufficiently small ($k < k_j$ and $v < v_j$), the prepayment falls further down below k driving the full payment to a level below total costs.¹⁴

To sum up, the larger the amount of investment in specific assets and the larger the marginal cost, the more likely it is that the prepayment will be greater than the specific-asset investment. In other words, whenever the supplier commits specific assets with a large non-salvageable value and production involves a high marginal cost, the seller will safeguard his investment by charging a large prepayment.

The previous remarks are also illustrated in figures 4 and 5. Each diagram in figure 4 describes how the equilibrium full payment (that is, the full payment that yields zero equilibrium expected profits) changes when v and k change, holding α constant. Figure 5 describes similar changes for the equilibrium prepayment.

When $\alpha = 1$ then we have shown that $p_j^{s,j} = v + k$ and $p_j^{s,j} = k$. This case is depicted in the last diagram of every figure.

We can see that when k and v are sufficiently large, it then becomes true that $p_j^{s,j} > v + k$ and $p_j^{s,j} > k$. All regions in figure 3 can be identified in figures 4 and 5.

Furthermore, figures 4 and 5 show how the full payment and the prepayment vary with α , holding v and k constant. As α increases, so that the amount by which the value of the prepayment to the seller is reduced decreases, both the full payment and the prepayment increase and converge to $v + k$ and k , respectively, whenever they lie below these levels. They decrease towards $v + k$ and k in cases where they lie above these levels.

We close this section with the following proposition which summarizes the main result.

Proposition 4. For any v, k, u_B^{w3} and α such that $v \in (0,1)$, $k \in (0,1)$, $u_B^{w3} \in (0,0.5)$, $\alpha \in (0,1)$ and $E\pi_s(p,v,k,\alpha) \geq 0$, there does not exist a contract of type $\wp 3$ that can support efficient exchange. The unique existing contract postulates that $p_2^{w3} > v$ so that exchange which is efficient according to the marginal-cost supply criterion may not take place.

7.4. Contracting Scheme $\wp 4$

Under this scheme, the supplier receives $p_1^{w4} - t^{w4}$ regardless of whether or not the buyer confirms the order. This implies that the supplier's expected profits are equal to

$$\begin{aligned} E\pi_s &= \int_0^{p_2} (p_1 - t^{w4} - k) dr + \int_{p_2}^1 (p_1 - t^{w4} + p_2 - v - k) dr \\ &= p_1 - t^{w4} - k + \int_{p_2}^1 (p_2 - v) dr \end{aligned}$$

which is equal to the supplier's expected profits under contracting scheme \wp from which the constant t^{w4} has been subtracted. Hence, using (5), we have that

$$E\pi_s = -\frac{1}{2}p_2^2 + vp_2 - (v+k) + \frac{1}{2} - u_B - t^{w4}. \quad (19)$$

Maximizing expected profits with respect to p_2 implies a value for p_2 equal to

$$p_2^{w4} = v. \quad (20)$$

Substituting into (19) and using (3), we get

$$\begin{aligned}
 E\pi_s(p; v, k, t^{D4}) &= -\frac{1}{2}v^2 + v^2 - (v+k) + \frac{1}{2} + p - \frac{1}{2}(1+v^2) - t^{D4} \\
 &= p - (v+k+t^{D4}).
 \end{aligned}$$

The assumption of competition in the supply side of the market implies that

$$p^{D4} = v + k + t^{D4}. \quad (21)$$

The prepayment is therefore equal to

$$p_1^{D4} = p^{D4} - p_2^{D4} = k + t^{D4}. \quad (22)$$

Contracting scheme $\rho 4$ always supports the efficient outcome: the buyer will confirm the order and exchange will take place whenever the realized value of the good is greater than or equal to v . This is to be contrasted with contract $\rho 2$ which may result to inefficient exchange taking place. The difference in the outcomes obtained under the two contracts hinges on the fact that under contract $\rho 4$ the transaction costs, t^{D4} , are incurred in all demand states, regardless of whether or not the order is confirmed. Thus, raising the amount of prepayment by t^{D4} [equation (22)] provides the supplier with a perfect hedge against the prospect of incurring transaction costs. There is no reason therefore why efficiency should be affected: changing p_2 does not alter the effects of t^{D4} on expected profits.

This is not the case, however, with contract $\rho 2$. The increase in p_1 by t^{D2} results to an inefficient outcome: since the transaction costs, t^{D2} , are incurred only when the buyer cancels the order, the supplier has an incentive to decrease p_2 since such a decrease lowers the probability that the order will be cancelled.

The following proposition summarizes the results under contract p^4 .

Proposition 5. *For any $v, k, u_a^{p^4}$ and t^{p^4} such that $v \in (0,1)$, $k \in (0,1)$, $u_a^{p^4} \in (0,0.5)$, $t^{p^4} \in (0,p_1^{p^4})$ and $E\pi_i(p;v,k,t^{p^4}) \geq 0$, there exists a unique contract of type p^4 which supports efficient exchange and postulates that*

$$\begin{aligned} p_1^{p^4} &= k + t^{p^4} \\ p_2^{p^4} &= v \\ p^{p^4} &= v + k + t^{p^4}. \end{aligned}$$

8. Prepayments as Opposed to Hostages: Some Critical Remarks on Williamson's Hostages as a Means to Support Exchange

Williamson's (1983) hostage model serves the same purpose as our prepayments model: restoring efficiency in the two-party market transaction m by safeguarding the seller's specific-asset investment against the buyer's opportunistic behaviour.

While prepayments are paid in advance, hostages are paid after the realization of the value of the good and only if the buyer cancels the order. Williamson, therefore, considers the following contracting scheme:

(H) The buyer places an order in period 1 and the producer makes the specific-asset investment. If the buyer confirms the order, he pays (in period 2) a unit price equal to p^H . If second-period delivery is cancelled, however, the buyer experiences a reduction in wealth of h .

It turns out that contract H supports efficient exchange as long as the hostage is set equal to the investment in specific assets. We derived the same result since the prepayment should be set equal to k if contract \varnothing is to support efficient exchange.

Nevertheless, our result has been derived by using, what we consider to be, a more appropriate solution procedure (this procedure is described on page 15). To obtain the desired result, Williamson sets the expected profits function [the equivalent of (4) with hostages] equal to zero and gets a set of (p, p_2) pairs (rather than a unique solution). Then, from among these pairs, he picks the pair that yields the efficient outcome.

This procedure has two drawbacks. First, the set of solutions is not sufficiently refined for analysis. Second, picking the efficient solution from a set just on the grounds that it is the desirable one, and without using an optimization procedure, does not really consolidate the position that contract H (or \varnothing) supports efficient exchange.

Williamson examines next a scenario where, if the order is cancelled, the buyer pays a hostage that he values in amount h with the seller valuing this hostage in amount αh , $\alpha \in (0,1)$. The analogous scenario in the case of prepayments is contract $\varnothing 1$.

We quote Williamson's brief analysis of this scenario:

Problems arise, however, if $h < k$ or $\alpha < 1$. The disadvantage, moreover, accrues entirely to the buyer - since the seller, by assumption, breaks even whatever contracting relation obtains. Thus although after the contract has been made, the buyer would prefer to offer a lesser-valued hostage and cares not whether the hostage is valued by the producer, at the time of the contract he will wish to assure the producer that a hostage of k for which the producer realizes full value ($\alpha = 1$) will be transferred in nonexchange states. Failure to make this commitment will result in an increase in the contract price. [Williamson (1983, p. 525); emphasis in bold added].

In relation to this quote we make the following remarks which serve to highlight the differences between Williamson's hostage model and our prepayments model.

1. When $\alpha = 1$, contract H yields a set of (p, p_2) pairs from which Williamson picks the one

that is in accordance with the marginal-cost supply criterion. However, it is not possible to follow the same procedure whenever $\alpha < 1$. The reason is that situations where $\alpha < 1$ give rise to *inefficient* outcomes, making it thus impossible to just pick the "right" pair from the set of inefficient (p, p_2) pairs. In other words, no "focal" point (like the efficient one) exists anymore.

These considerations imply that it is not possible to obtain a unique solution in Williamson's model, as far as contract H is concerned. Nevertheless, the solution procedure that we adopted gave us a unique solution for contract $\rho 1$ (that is, the contract that is analogous to contract H) as well as for contracts $\rho 2$, $\rho 3$ and $\rho 4$ (which are not considered by Williamson).

2. It is because of the inability to get an explicit solution that Williamson does not examine the relation between the values of α and k . He mentions that "problems arise, however, if $k < k$ or $\alpha < 1$."

We have shown, however, that the equilibrium value of the prepayment relative to the amount of the investment in specific assets (k) critically depends on the value of α . This is true for both contracts $\rho 1$ and $\rho 3$. Furthermore, it may well be the case that the equilibrium prepayment is greater than the investment in specific assets.

3. Although Williamson recognizes that "problems arise...if $\alpha < 1$ ", he does not say anything about the type of inefficiency that a value of α which is less than unity gives rise to.

In contrast, we have shown that whether p_2 is greater or less than v depends on the type of contract considered. Moreover, there exist contracts that support efficient outcomes even though $\alpha < 1$. This is the case of contract $\rho 4$ where the full value of the prepayment to the seller is reduced by a constant amount under all possible product-value realizations.

4. Williamson contends that the contract price (or the full payment, p) will increase (above

$v + k$) as a consequence of the seller receiving a hostage that is valued by him in amount αh , $\alpha < 1$.

We found that the size of the equilibrium full payment depends on the type of the contract under consideration. For instance, when the full value of the prepayment to the seller is reduced by a constant amount and only whenever the order is cancelled (contract $\rho 2$), the full payment remains unchanged and is still equal to total costs, $v + k$. It may be less than total costs in some situations under contract $\rho 3$.

We hope to have shown that Williamson's hostage model, which forms part of his pathbreaking work on transaction cost economics, is not entirely adequate. It is hoped that the contribution in this paper has extended and exhausted Williamson's argument.

9. Concluding Remarks

One of the central points of this paper concerns asset specificity. The seller invests in assets that are put in place only for the prospect of selling an amount of the product to a particular customer. Premature termination of the contract by the buyer would leave the supplier with a large specialized capacity to be redeployed at a greatly reduced alternative value.

Requiring buyers to prepay would check the hazard of the seller having to dispose of a large capacity only at distress prices and would support efficient exchange. The use of prepayments, therefore, to support exchange is economically important.

Nevertheless, one needs to consider expropriation hazards: the supplier may contrive to renege on the promised investment and expropriate the prepayment.

In consideration of these expropriation hazards, the buyer and the seller may seek to devise alternative contractual relations such as reciprocal trading.

Appendix

A1. Seller's Expected Profits under Contract $\varphi 1$. These are given by

$$\begin{aligned} E\pi_s &= \int_0^{p_1} (\alpha p_1 - k) dr + \int_{p_1}^1 (p_1 + p_2 - v - k) dr \\ &= p_2(\alpha p_1 - k) + (1 - p_2)(p_2 + p_1 - v - k) \\ &= [p_2(\alpha - 1) + 1](p - p_2) + (1 - p_2)(p_2 - v) - k \\ &= [p_2(\alpha - 1) + 1] \left[\frac{1}{2}(1 + p_2^2) - u_g - p_2 \right] + (1 - p_2)(p_2 - v) - k \\ &= \frac{1}{2}(\alpha - 1)p_2^3 + \frac{1}{2}(1 - 2\alpha)p_2^2 + \frac{1}{2}[(\alpha - 1)(1 - 2u_g) + 2v]p_2 + \frac{1}{2} - u_g - (v + k). \end{aligned}$$

A2. Restrictions on the Values of α , v and u_g Implied by the Choice of p_1^{p1} . The two solutions in question are given by the following equation:

$$(p_2)_s = \frac{2\alpha - 1 \pm \sqrt{(1 - 2\alpha)^2 - 3(\alpha - 1)[(\alpha - 1)(1 - 2u_g) + 2v]}}{3(\alpha - 1)}.$$

The actual sign of p_2 depends on that of $(\alpha - 1)(1 - 2u_g) + 2v$. We consider the following cases.

Case 1: $0 < \alpha < 1/2$ and $(\alpha - 1)(1 - 2u_g) + 2v < 0$.

These relations imply that $(p_2)_s > 1$.

Case 2: $0 < \alpha < 1/2$ and $(\alpha - 1)(1 - 2u_g) + 2v > 0$.

Then we have that $(p_2)_s < 0$.

Case 3: $1/2 \leq \alpha < 1$.

This again implies that $(p_2)_\alpha < 0$.

Case 4: $0 < \alpha \leq 1/2$.

This implies that $(p_2)_\alpha > 0$. Furthermore, if $(\alpha - 1)(1 - 2u_B) + 2v < 1 + \alpha$, it is also true that $(p_2)_\alpha < 1$.

Case 5: $1/2 < \alpha < 1$ and $(\alpha - 1)(1 - 2u_B) + 2v > 0$.

These inequalities imply that $(p_2)_\alpha > 0$. If $(\alpha - 1)(1 - 2u_B) + 2v < 1 + \alpha$ is also true, then we have once more that $(p_2)_\alpha < 1$.

Consideration of the above cases lead us to conclude that if $0 < (\alpha - 1)(1 - 2u_B) + 2v < 1 + \alpha$ then $(p_2)_\alpha \in (0, 1)$ for any $\alpha \in (0, 1)$.

A3. *Seller's Expected Profits under Contract $\phi 2$.* From section 7.2, we have that

$$\begin{aligned} E\pi_B &= \int_0^{p_1} (p_1 - t^{B2} - k) dt + \int_{p_1}^1 (p_1 + p_2 - v - k) dt \\ &= p_1(p_1 - t^{B2} - k) + (1 - p_1)(p_1 + p_2 - v - k) \\ &= (p - p_2) - k - p_2 t^{B2} + (1 - p_2)(p_2 - v) \\ &= \left[\frac{1}{2}(1 + p_2^2) - u_B - p_1 \right] - k - p_2 t^{B2} + p_2 - v - p_2^2 + p_2 v \\ &= -\frac{1}{2} p_2^2 + (v - t^{B2}) + \frac{1}{2} - u_B - (v + k). \end{aligned}$$

A4. *Seller's Expected Profits under Contract $\phi 3$.* From section 7.3, we have

$$\begin{aligned}
E\pi_g &= \int_0^{p_1} (\alpha p_1 - k) dr + \int_{p_1}^1 (\alpha p_1 + p_2 - v - k) dr \\
&= p_2 (\alpha p_1 - k) + (1 - p_2) (p_2 + \alpha p_1 - v - k) \\
&= \alpha (p - p_2) + p_2 - (v + k) - p_2^2 + v p_2 \\
&= \alpha \left[\frac{1}{2} (1 + p_2^2) - u_B - p_2 \right] + p_2 - (v + k) - p_2^2 + v p_2 \\
&= \left(\frac{\alpha}{2} - 1 \right) p_2^2 - (\alpha - 1 - v) p_2 + \frac{\alpha}{2} - \alpha u_B - (v + k).
\end{aligned}$$

A5. *Computation of $E\pi_s(p, v, k, \alpha)$ and p^{s1} .* Substituting for p_2^{s1} [using (16)] into (15) and using (3), we get

$$\begin{aligned}
E\pi_s(p, v, k, \alpha) &= \left(\frac{\alpha}{2} - 1 \right) \frac{[v + (1 - \alpha)]^2}{(2 - \alpha)^2} + \frac{[v + (1 - \alpha)]^2}{2 - \alpha} + \frac{\alpha}{2} - \alpha \left\{ -p + \frac{1}{2} + \frac{1}{2} \frac{[v + (1 - \alpha)]^2}{(2 - \alpha)^2} \right\} - (v + k) \\
&= - \frac{[v + (1 - \alpha)]^2}{(2 - \alpha)^2} + \frac{[v + (1 - \alpha)]^2}{2 - \alpha} + \alpha p - (v + k) \\
&= - \frac{(\alpha - 1)[v + (1 - \alpha)]^2}{(2 - \alpha)^2} + \alpha p - (v + k).
\end{aligned}$$

Competition in the supply side of the market implies that $E\pi_s(p, v, k, \alpha)$ should be set equal to zero. This gives a solution for p equal to

$$p^{s1} = \frac{(\alpha - 1)[v + (1 - \alpha)]^2}{\alpha(2 - \alpha)^2} + \frac{v + k}{\alpha}.$$

Notes

1. We define an "efficient" exchange as one in which the product is exchanged whenever its value to the buyer is greater than or equal to its marginal cost of production.

2. Prager (1990) presents some empirical evidence on the role of reputation effects in weakening the ability of franchise winners to engage in *ex post* opportunistic behaviour by reneging on the promises that they made in order to win the franchise contract. The econometric results offer weak support for the existence of reputation effects as a constraint on firm behaviour. If reputation effects exist, they should be stronger for firms interested in expanding into new markets than for firms with no expansion plans. Furthermore, those communities with the best information should benefit the most from reputation effects and, other things being equal, should experience the smallest deviation from contractual promises.

3. An interesting example is found in Marks (1990) about John Palmisano, a former US Environmental Protection Agency (EPA) employee who is the founder of Air Emissions Reduction Exchange (Washington, DC). Palmisano is a pollution consultant and trader of emissions-reduction credits (ERC). ERCs are an EPA creation that allows companies to earn credits by voluntarily cutting pollution below legal limits. The company can bank the credits to use when it expands, or it can sell the credits to another company in the same area. ERCs were intended as a free-market approach to the economic aspects of environmental pollution problems. The value of ERCs has increased at a rate of about 30% per year since 1983. A credit to emit oxides of nitrogen for a year can trade for \$9000 a ton. Palmisano charges an up-front fee and about 20% of the savings he arranges when he finds and brings together in

a contractual exchange buyers and sellers of ERCs.

4. In describing the model, we will adopt Williamson's (1983) framework. Of course, it is prepayments rather than hostages that we are interested in.

5. Williamson (1985, p. 54) argues that rather than distinguishing between fixed and variable costs, more relevant to the study of contracting is whether assets are redeployable or not [Klein and Leffler (1981)]. Many assets that accountants regard as fixed are in fact redeployable; for example, centrally located general-purpose buildings and equipment. Other costs that accountants treat as variable often have a large non-salvageable part, firm-specific human capital being an illustration.

Figure 1 helps to make the distinction. Costs are distinguished as to fixed (F) and variable (V) parts. But they are further classified as to the degree of specificity, of which only two kinds are recognized: wholly specific (k) and non-specific (v). The region labelled " k " at the bottom of the figure is the troublesome one for purposes of contracting.

6. Superscripts will be denoting equilibrium values.

7. The algebra is shown in appendix A1.

8. Appendix A2 explains the restrictions on the values of the parameters of the model that the choice of the acceptable root imposes.

9. Some steps of the algebra are presented in appendix A3.

10. This is to be contrasted with the result under contract $\rho 1$ where the full payment increases and becomes greater than total costs ($v + k$). If the seller were to adopt the same pricing strategy under contract $\rho 1$ as under $\rho 2$ then he could become constrained to charge a negative period-2 price ($p_2^{\rho 1} < 0$). To see this, adopting the $\rho 2$ -contract pricing strategy for the case of contract $\rho 1$ implies that

$$\begin{aligned} p_1^{\rho 1} &= \frac{k}{\alpha} \\ p^{\rho 1} &= v + k \\ p_2^{\rho 1} &= v + \frac{\alpha - 1}{\alpha} k \end{aligned}$$

if the seller is to receive a prepayment which he values at k in cancellation states and the full payment is to be equal to total costs. Nevertheless, it may be the case that $p_2^{\rho 1} < 0$. Table 1, for example, can be used to show that $p_2^{\rho 1} < 0$ for any α such that $\alpha > 0$ and $\alpha < k/(v + k) = 0.3125$. Hence, the seller finds it optimal to increase the prepayment above k by an amount smaller than the one that adoption of the $\rho 2$ -contract pricing strategy would imply and, at the same time, increase the full payment and the period-2 payment above $v + k$ and v , respectively, in order to make up for the limited increase in the prepayment.

11. The algebra is presented in appendix A4.

12. Some steps of the algebra are shown in appendix A5.

13. In figure 3, k_1 , k_2 , v_1 and v_2 are equal to the following:

$$k_1 = \frac{(1-\alpha)^2}{(2-\alpha)^2}$$

$$k_2 = \frac{1}{(2-\alpha)^2}$$

$$v_1 = \frac{1}{2} \{ 1 + (1-\alpha)^2 - \sqrt{1 - (1-\alpha)^2 + 4(2-\alpha)^2 k} \}$$

$$v_2 = 1 - (2-\alpha)\sqrt{k}.$$

14. To avoid confusion, it should be stressed that we always compare the *equilibrium* full payment against total costs. Hence, although it may be the case that $p^{*1} < v + k$, it does not follow that the equilibrium expected profits are negative. Equilibrium expected profits will always be zero since p^{*1} solves the equation $E\pi_2(p; v, k, \alpha) = 0$.

<i>F</i>	<i>V</i>
<i>v</i>	<i>v</i>
<i>k</i>	<i>k</i>

Figure 1. *Cost Distinctions*

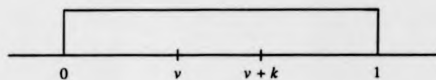


Figure 2. *Demand Distribution and Costs of Supply*

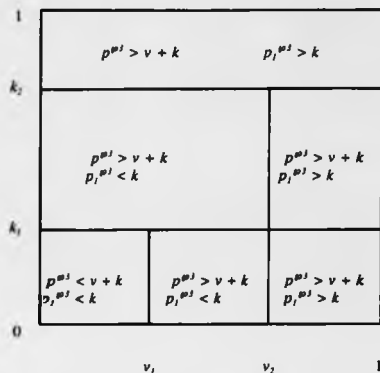


Figure 3. *The Relationship Between the Equilibrium Contract- $\phi 3$ Full Payment and Total Costs, and Between the Equilibrium Contract- $\phi 3$ Prepayment and the Specific-Asset Investment, Given a Value for α*

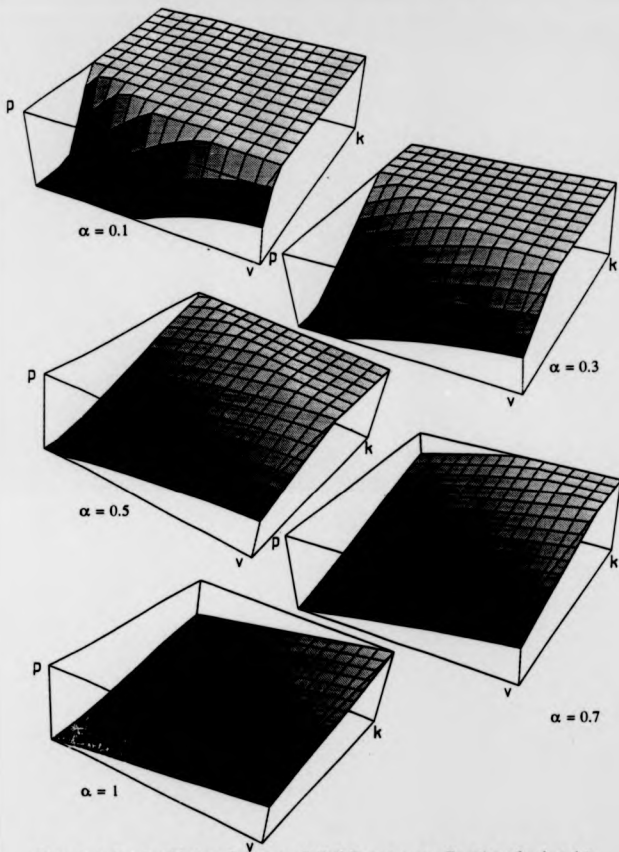


Figure 4. The Equilibrium Contract- ϕ_3 Full Payment as a Function of v , k and α .

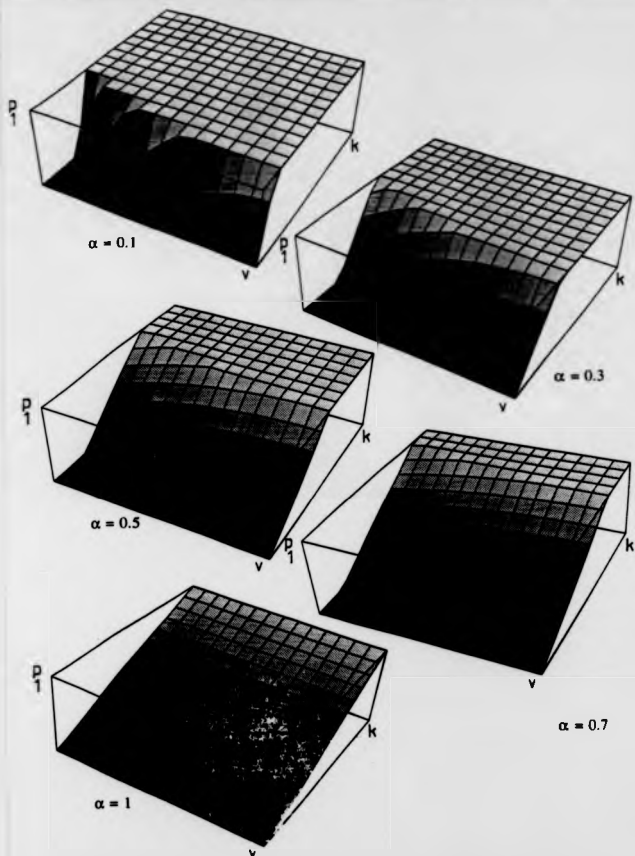


Figure 5. The Equilibrium Contract ω 3 Pr. payment as a Function of v , k and α

Table 1

Numerical Solutions for Contract $\phi 1$ When $v = 0.33$ and $k = 0.15$

α	$p^{p'}$	$p_i^{p'}$	$p_t^{p'}$	$u_g^{p'}$
0.1	0.559612	0.182881	0.376731	0.0113507
0.2	0.553018	0.167668	0.38535	0.0212296
0.3	0.543485	0.159914	0.383571	0.0300782
0.4	0.533258	0.155595	0.377663	0.0380566
0.5	0.523119	0.153145	0.369974	0.0453211
0.6	0.513395	0.151754	0.361641	0.0519972
0.7	0.504212	0.150961	0.353251	0.0581816
0.8	0.495599	0.150497	0.345102	0.0639486
0.9	0.487541	0.150208	0.337333	0.0693561

3 AN OPTIMAL SALES STRATEGY FOR A MONOPOLIST OF A NEW DURABLE GOOD: CONTRACTS AND PREPAYMENTS

1. Introduction

In this chapter we will analyze a monopolist's pricing decision of a new durable good under conditions of cost uncertainty. Consider a firm that has developed a new product which is going to be produced and launched into the market only if the unit production cost (assumed to be constant) is revealed to be sufficiently low. This feature appears explicitly in the model by letting the unit production cost be a random variable whose value becomes common knowledge *after* the monopolist has spent money on R&D and determined the expected profit-maximizing pricing policy. Suppose further that each buyer will want one unit of the good and that there is a distribution of consumer types indexed by their valuation of the good. It is assumed that the monopolist cannot identify consumers by their valuation. Consumers know with certainty the true value of the unit cost when it is revealed and the pricing policy of the firm.

The fact that the monopolist has already spent a non-recoverable amount of money on R&D makes him worry about the outcome of his investment. In other words, if the value of the unit production cost turns out to be too high for production to be profitable, the monopolist will not be able to cover his sunk costs simply because production will not take place. It is reasonable to argue, therefore, that the monopolist would like to insure his assets

against this possibility. How could he achieve that?

The answer to this question hinges on the assumption that there are different consumer types. The higher the type of a consumer (the higher his product valuation), the more this type's utility will be reduced by purchasing and consuming the durable good later rather than earlier. It is this feature that the monopolist tries to exploit in order to insure his assets against the possibility of non-production. The monopolist can do that by charging consumer types with a sufficiently high valuation for the product a non-refundable prepayment payable before the unit production cost becomes common knowledge and, at the same time promising them to be the first to get the good if it is produced, in which case, they will also pay a unit price (payable upon delivery). For these types, the expected benefits from getting the good sooner than others outweigh the risk of losing the prepayment and not getting the good in the case where the unit cost turns out to be too high for production to be profitable.

Finally, those consumers for whom the expected benefits from early delivery do not outweigh the risk of losing the prepayment will choose not to pay the prepayment and, consequently, they will be able to buy the good (if it is produced) only after those who have prepaid are served.

The model is formulated in discrete time. In period 0, the monopolist truthfully announces that he has spent an amount of money (representing the sunk cost) on the development of a new durable good and informs consumers about the existence of cost uncertainty. Furthermore, he determines the expected profit-maximizing pricing policy and he receives prepayments from those consumers willing to prepay in order to be the first to get the good if it is produced. Then, in the beginning of period 1, the actual value of the unit cost is revealed and the monopolist decides whether or not to go on with production. If the unit cost is revealed to be higher than the expected profit-maximizing period-1 price so that production

does not take place, those consumers who prepaid lose their money which now serves to cover (part of) the monopolist's sunk costs whereas if production does take place, these consumers get the durable good in period 1 by paying the appropriate unit price which has been determined in period 0. Consumers who have not prepaid form a pool of customers that must wait until period 2 in order to purchase one unit of the product.

After describing the results of some previous research sharing some common elements with the present work, we will indicate in section 3 that the period-2 price will be equal to the actual value of the marginal cost. We will also discuss how the Coase conjecture [Coase (1972)] is related to this exercise.

In section 4 the behaviour of consumers and the monopolist is analyzed. Given the prepayment and the period-1 and period-2 prices, the marginal consumer is characterized by the fact that he is indifferent between paying the prepayment in period 0 (with the consequence of getting the good in period 1 if it is produced) and just waiting until period 2. Then the expected profit-maximizing pricing policy of the monopolist is determined. Given the marginal consumer, the monopolist determines the amounts of prepayments and period-1 price that maximize his expected profits.

In section 5 the welfare implications of the expected profit-maximizing pricing policy are examined and we compare the expected welfare at the expected profit-maximizing solution with the expected welfare under two alternative situations. In section 6 the role of the prepayment as a facilitating device to give the monopolist access to financial markets is illustrated. Finally, some concluding remarks are made in the last section.

2. Previous Relevant Research

Several authors have examined how a monopolist may use uncertainty in *demand* (rather than cost) in order to exploit demand differences across customers. Tschirhart and Jen (1979) and Harris and Raviv (1981) show that by creating a priority pricing system, high-valuation customers can be induced to pay more for a unit than low valuation customers as long as paying a premium guarantees priority treatment in case of random quantity rationing.

In particular, Tschirhart and Jen (1979) examine the behaviour of a monopoly offering interruptible service. The idea is that service to consumers is interrupted in a predetermined order when available supply falls short of demand (shortages occur because of the stochastic nature of demand). The monopolist divides the customers into classes and then determines the order in which service to these classes is interrupted. Classes receiving a lower priority of service are charged lower prices. The monopolist maximizes profits by interrupting the lowest-priced class first, the second lowest-priced class second, etc.

Chao and Wilson's (1987) work is similar. They also examine priority service offering customers a menu of contingent contracts for distribution of scarce supplies. Priority service can be implemented in the form of the sale of priority points or the provision of compensatory insurance. They show that priority service with only a few priority classes suffices to achieve most efficiency gains.

Harris and Raviv (1981) examine the optimality of the priority pricing scheme from the point of view of a monopolist who faces a potential demand that exceeds capacity and who does not have full information about the reservation price of each buyer. In the absence of such full information, it is possible to discriminate by charging some buyers higher prices in exchange for higher priority access to the product. Each buyer chooses a priority price which he is willing to pay. Buyers are then ranked by the priority price they choose, that is, buyers choosing higher priority prices are assigned higher rank.

Finally, Png (1991) analyzes how a seller should price production capacity to heterogeneous buyers with private information about their valuations. One option for the seller is to discriminate between customers by offering prices that decline over time. As the price declines, so does the likelihood of obtaining a unit of capacity. The uncertainty of obtaining service at a lower price compels customers with a higher valuation to buy in advance. However, the higher valuation customers' willingness to gamble on a price cut in the following period limits the seller's ability to extract consumer surplus. The seller can mitigate this problem by offering most-favoured-customer (MFC) status protection, which guarantees that customers who buy early will benefit from subsequent reductions in price. When choosing between MFC and price discrimination, the seller will prefer MFC protection when capacity is large because MFC supports a higher price for advance sales.

In the previous papers, a monopolist engages in price discrimination by choosing a pricing scheme which causes high-valuation customers to pay a high price for assured delivery and low-valuation customers to pay a low price for risky delivery.

Our setting, however, differs in a number of points. The introduction of *cost* uncertainty and, therefore, the possibility of non-production imply that *both* the high- and low-valuation customers face risky delivery. Furthermore, the seller is not imminently interested in price discrimination in the sense that he does not offer a "price-probability of service" schedule just for the sake of inducing customers to self-select along this schedule and, thereby, extracting their surplus. Rather, his priority is to *obtain some revenue in those cases where the product turns out to be too costly to be produced*. This is the novel idea of our model which gives rise to the use of prepayments as a means to obtain this revenue. Charging prepayments in order to obtain some revenue in the case where the product turns out to be unprofitable comes at the cost of having to allocate sales between two periods with the consequence of receiving

zero profits from the last-period sales (we will demonstrate that the price in this last period of sales should be equal to marginal cost). This is unavoidable since only if the high-valuation customers are guaranteed to get the product sooner than others will they then be willing to prepay.

Another characteristic of our model not present in the models discussed above is that although high-valuation customers have the right to purchase the product earlier than others as long as they have prepaid in period 0, they will not exercise this right if the unit cost turns out to be sufficiently low. Then, some period-1 customers will enjoy a higher (ex post) surplus if they purchase the product in period 2 at a price equal to marginal cost even though by doing so they are not refunded the prepayment.

Related to our work are also the papers by Stokey (1979), and Ireland and Stoneman (1985, 1986). These papers share the same idea with ours in that investigate situations in which some purchasers of a product (or a technology) derive greater gross surplus by purchasing and using the product earlier than others or earlier than later in time. Nevertheless, the element of cost uncertainty which motivates the use of prepayments in our model is not present in these models. Stokey examines a situation where buyers' reservation prices depend on time. She showed that when production costs are positive and declining and buyers differ considerably in their costs from delayed purchases then intertemporal price discrimination is optimal. Ireland and Stoneman (1985) focus on a situation where each buyer's reservation price depends on the number of sales of the product to date-of-purchase. Buyers with perfect foresight over the time path of prices are willing to pay premia for early delivery of the product. This allows the producer to price discriminate over time charging different buyers, different prices at different times. Ireland and Stoneman (1986) consider the adoption decision of a new technology by buyers who are indexed in decreasing order of the size of the service

flow they obtain from the use of the technology until obsolescence. A potential purchaser will acquire one unit of the technology at a particular time if the technology is profitable and if profitability is not expected to increase. Buyers with a larger index will purchase the technology earlier than others.

3. The Period-2 Price and the Coase Conjecture

The best way to start this discussion is to examine the behaviour of a monopolistic seller of a durable good, as discussed by Coase (1972). Assume that the consumers can enjoy a certain good which is infinitely durable and suppose that both the monopolist and the consumers are infinitely lived. The price at which consumers are willing to buy the good today then depends on their expectation of the price at which they will be able to buy it tomorrow, because today's purchases are a substitute for tomorrow's. In a setting like the above, the monopolist is constrained by the consumers' rational beliefs that he will flood the market, after he has made his initial sales and that the price will be lower in the future. As a matter of fact the monopolist does that since, by selling more of the good, he can acquire more money and improve his position. Then consumers do not buy and wait for the day when the monopolist will cut his price. The *Coase Conjecture*, proved formally by Bulow (1982) and Stokey (1981) for particular demand functions and equilibria and Gul, Sonnenschein, and Wilson (1986) for more general demand structures, states that when the price adjustments become more and more frequent the monopolist's profit converges to zero. All trade takes place almost instantaneously, at prices close to marginal cost. The reason is that, in equilibrium, consumers expect the monopolist to charge prices close to the competitive price at any future instant and, as they can wait for the next offer without delay cost, they cannot be induced to

accept higher prices. Thus, the monopolist ends up charging prices close to the competitive prices, fulfilling the consumers' expectations.

There have been several attempts to show that a monopolistic seller of a durable good will use intertemporal price discrimination [an overview of them is given by Tirole (1988, p. 85-86)]. In our case, we would expect the monopolist to sell at decreasing prices at successive dates. The key assumption is that some consumers are ready to pay a higher price in order to get the good earlier. Since high-valuation customers benefit more by consuming the good sooner than later, the knowledge that other buyers may get a better price, *later*, does not prevent the high-valuation customers from paying the higher price. The high-valuation customers' expected surplus from prepaying will be greater than their expected surplus from purchasing the product in the last period. This argument relies entirely on the presence of time preference. The existence of time preference enables the monopolist to sustain a period-1 price greater than marginal cost. This is not going to be true, however, for period 2. Period-2 customers form a pool of consumer types that do not attach high value to advance delivery and they can wait without considerable delay costs. If we then think of period 2 as a time interval which consists of an *infinitely* large number of subperiods so that the monopolist cannot credibly charge and sustain a particular price until the next subperiod then the Coase conjecture implies that the period-2 price should be equal to marginal cost.

This result can be reinforced by assuming that the monopolist's product is threatened with imitation by equally-efficient firms which consider entering the market in period 2. The prospect of potential competition can keep period-2 price down to marginal cost.

4. Consumers' Behaviour and the Expected Profit-Maximizing Sales Strategy

4.1. Consumers' Behaviour

The population consists of a continuum of consumer types, indexed by t . The index t is a measure of each type's valuation of the good and is assumed to be described by a uniform distribution with support on the interval $[0,1]$.

If a consumer decides to buy some of the durable good in either period 1 or period 2, he will buy one unit of it. If his index is t and he buys one unit in period 1, he will derive a utility

$$U(1,t) = \gamma t$$

from its consumption where γ captures a notion of time preference and $\gamma > 1$.

If the consumer buys one unit in period 2, the utility that he will derive from the consumption of this unit is

$$U(2,t) = t.$$

This formulation implies that all individuals prefer to have the good sooner rather than later, that an individual with a higher t derives a greater amount of utility from the consumption of one unit of the good regardless of which period he purchased it in and that those who value the good more highly benefit less by consuming it later. That is, for every t in $[0,1]$ we have

$$U(1,t) \geq U(2,t)$$

$$U_i(1,t) > 0, \quad U_i(2,t) > 0$$

$$U_i(1,t) > U_i(2,t).$$

The validity of the last inequality allows the monopolist to exploit high-valuation customers by making them pay a prepayment in exchange for priority access to the product.

Consider a price schedule or, in other words, a period-0 contract $\{p_0, p_1, p_2\}$ where p_0 denotes the prepayment that has to be paid in period 0 and before the unit production cost becomes common knowledge. p_1 denotes the period-1 price and p_2 is the period-2 price contingent on the value of the unit cost as it will be realized in the beginning of period 1. We will assume that the unit production cost, c , is constant and is regarded by the monopolist and the consumers as a random variable which, at the time of contract signing, is described by a uniform distribution with support on the interval $[0,1]$.

If the unit cost is revealed to be less than or equal to the period-1 unit price so that the new product is profitable to produce and if a consumer of type t has already prepaid p_0 in period 0, then he will receive one unit of the good in period-1 at an additional price of p_1 and his ex post net welfare gain will be as follows:

$$U(1,t) - p_0 - p_1 - \gamma t - p_0 - p_1.$$

If this consumer has not prepaid p_0 and the product is profitable, he can buy one unit of the durable good in period 2 at a price of p_2 (equal to the realized marginal cost) deriving an ex post net welfare gain of

$$U(2,t) - p_2 - t - p_2.$$

Finally, if the new durable good is not profitable, a consumer of type t who has prepaid p_0 does not consume anything in period 1 and his ex post net welfare gain is $-p_0$ whereas if he has not prepaid, his ex post net welfare gain is zero.

Customers who have already prepaid p_0 in period 0 and expect to get the product in period 1 may reckon (after c becomes common knowledge in the beginning of period 1) that their interest actually lies in purchasing the product in period 2 rather than getting delivery in period 1 and paying a price of p_1 . If these customer types decide to behave in this way, they will have to lose the prepayment that they have already paid since it is a feature of the period-0 contracts that prepayments cannot be refunded under any circumstances.

The reason why customers who have prepaid may decide to behave in the above way is that the prepayment represents a non-recoverable cost for those consumers who have already paid this amount. This implies that, after the value of c becomes common knowledge, these consumers will seek to maximize their utility from that point on and the decision as to the most appropriate behaviour to achieve this maximization will depend on the realized value of c .

To make this point clear, note that for each type t who has prepaid p_0 there exists a critical cost value, say $c(t)$, such that if $c(t)$ happens to be the actual cost realization in the beginning of period 1 then the following will be true (as long as $c(t) \leq p_1$ so that the product is profitable):

$$u^t - p_1 = t - c(t).$$

In other words, consumer type t will be indifferent (ex post) between purchasing one unit of the good in period 1 (as he was intending to, before the unit cost was revealed to be equal

to $c(t)$], and renegeing and purchasing this unit in period 2 at a price of $p_2 = c(t)$.

Consumer type t will prefer (ex post) to purchase the product in period 1 if the value of the realized cost (say c_R) is greater than $c(t)$ since then type t 's ex post net benefits from period-1 purchase and consumption of the good will be greater than the corresponding period-2 net benefits:

$$yt - p_1 = t - c(t) > t - c_R$$

If, on the other hand, $c_R < c(t)$ then

$$yt - p_1 = t - c(t) < t - c_R$$

and type t will renege and buy in period 2 since his ex post period-2 net benefits are greater than his ex post period-1 net benefits.

Now we can proceed with the ex ante determination of the marginal consumer. The marginal consumer is defined as that type which is indifferent between prepaying in period 0 and waiting until period 2 in order to purchase the product at marginal cost. The marginal consumer type is determined before the value of cost becomes common knowledge since prepayments have to be paid in period 0.

It should be made clear that renegeing opportunities, if any, can be exploited only *after* c becomes known. This is to be distinguished from the procedure of determining the marginal consumer which takes place *before* c becomes known. It is, therefore, possible that the *ex ante* determined marginal consumer is no longer indifferent as to the time of purchase after the unit cost becomes common knowledge.

In the remainder of the paper we will assume that the expected net benefits that the

marginal consumer (denoted by T) would enjoy if he were to purchase the product in period 2 are non-negative for every cost realization in $[0, p_1]$. In other words, whenever the product is profitable, the marginal consumer would derive non-negative expected net benefits if he chose to purchase one unit of it in period 2. Given that the period-2 price is equal to marginal cost this assumption implies that the following is true:

$$\int_0^{p_1} (T - c) dc \geq 0.$$

The validity of this inequality presupposes that $T \geq p_1$ since, otherwise, the marginal consumer would not purchase the good in period 2 for any cost realization in $(T, p_1]$ and the upper limit of the integral in the previous inequality would be T . Furthermore, since $T \in [0, 1]$, we have to assume that $p_1 \leq 1$.

4.1.1. Determination of the Marginal Consumer, T , Given a Period-0 Contract (p_0, p_1, p_2)

The marginal consumer T is indifferent between prepaying and buying the product in period 2. In other words, his expected net benefits from prepaying are equal to those from not prepaying. When the marginal consumer computes his expected net benefits from prepaying, he must also take into account the possibility that, after observing the cost value in the beginning of period 1, he may change his mind and purchase the product in period 2. This will happen if the realized cost is less than $c(T)$ (i.e., less than the critical cost value for the marginal consumer). If it is greater than $c(T)$ then the marginal consumer will purchase the product in period 1. Hence, the equation that characterizes the marginal consumer is given

by the following:

$$-p_0 + \int_{c(T)}^{p_1} (\gamma T - p_1) dc + \int_0^{c(T)} (T - c) dc = \int_0^{p_1} (T - c) dc \quad (1)$$

where $c(T) = p_1 - (\gamma - 1)T$.

By rearranging equation (1) and integrating, we get the following:

$$(\gamma - 1)T[p_1 - c(T)] - \frac{1}{2}[p_1 - c(T)]^2 = p_0. \quad (2)$$

From the definition of $c(T)$, it is true that

$$p_1 - c(T) = (\gamma - 1)T. \quad (3)$$

Then (2) becomes as follows:

$$\frac{1}{2}(\gamma - 1)^2 T^2 = p_0. \quad (4)$$

Hence, all consumer types in the interval $[T, 1]$ will prepay since their expected net benefits from paying the period-1 price exceed their expected net benefits from purchasing the product in period 2 by at least the amount of the prepayment. That is,

$$\frac{1}{2}(\gamma - 1)^2 t^2 \geq p_0. \quad (5)$$

On the other hand, all types in $[0, T)$ will wait until period 2 in order to purchase one unit of the product.

Let's denote the left-hand side of inequality (5) by $S(\gamma, t)$:

$$S(\gamma, t) = \frac{1}{2}(\gamma - 1)^2 t^2 \geq 0.$$

The function $S(\gamma, t)$ has the following properties:

$$S(\gamma, 0) = 0$$

$$S(\gamma, 1) = \frac{1}{2}(\gamma - 1)^2 > 0$$

$$\frac{\partial S}{\partial t} = (\gamma - 1)^2 t \geq 0$$

$$\frac{\partial^2 S}{\partial t^2} = (\gamma - 1)^2 > 0.$$

Using these relations we can illustrate inequality (5) in figure 1.

As $S(\gamma, t)$ is increasing and strictly convex on the interval $[0, 1]$, there exists a marginal consumer type, T , such that equality (4) is satisfied.

4.1.2. Effects of Changes in p_0 and γ on the Value of the Marginal Consumer

The top diagram in figure 2 shows that an increase in p_0 increases the marginal consumer type and, therefore, decreases the number of customers who prepay since now fewer consumers

have a surplus $S(\gamma, t)$ high enough to warrant the higher prepayment.

The bottom diagram in the same figure illustrates the effect of a higher γ or a stronger desire to get one unit of the durable good sooner rather than later. An increase in γ results to a pivoting (around the origin) towards the left of the curve representing the function S . This movement represents an increase in the surplus of each consumer type. This implies that a higher number of consumers choose to prepay p_0 and the marginal consumer is represented by a lower type.

4.2. Determination of the Expected Profit-Maximizing Pricing Policy Given the Equation for the Marginal Consumer T

We will assume that the monopolist puts extra value on having some revenue before the actual value of the unit cost is realized rather than after. This assumption is made explicit in the model by introducing a discount factor, β , which applies to the monopolist's decision problem. It is assumed that $\beta \in (0, 1)$.

The monopolist must take into account the existence of *ex post* reneging possibilities in determining the prepayment and the period-1 price that maximize his expected profits. Whether or not reneging of those consumers who have prepaid will occur depends on the actual value of c .

If c takes a value in the interval $[c(T), p_1]$ then the following will be true *ex post*:

$$\gamma T - p_1 \geq T - c,$$

where $c_r \in [c(T), p_1]$ is the realized cost value. Equivalently,

$$(\gamma - 1)T \geq p_1 - c_r.$$

Hence, whenever the actual cost value lies in the interval $[c(T), p_1]$ then no consumer type in $[T, 1]$ - that is, no consumer type who has prepaid p_0 - will renege and buy in period 2. In this case, therefore, the monopolist's period-1 profits (ignoring receipts from prepayments) are equal to

$$\beta(1 - T)(p_1 - c_r).$$

Suppose now that the actual cost, say c_R , lies in the interval $[0, c(T)]$. In this case, the marginal consumer type T will prefer to renege and buy one unit of the product in period 2 instead of period 1 since for this type the following is true *ex post*:

$$(\gamma - 1)T < p_1 - c_R.$$

Furthermore, not only is T going to renege and purchase in period 2 but all types in the interval (T, T_R) will renege as well, where T_R is the new *ex post* marginal consumer type. This implies that only $1 - T_R$ consumers will now buy in period 1 instead of the $1 - T$ that had been willing to do so before c was revealed to be equal to c_R . The value of T_R is monotonically decreasing in c_R and satisfies the following equation:

$$\gamma T_R - p_1 = T_R - c_R.$$

Hence, the ex post marginal consumer type T_R is equal to

$$T_R = \frac{p_1 - c_R}{\gamma - 1}.$$

The monopolist's period-1 profits, therefore, for this case are equal to

$$\beta(1 - T_R)(p_1 - c_R).$$

Given the above considerations as well as that

- (1) the monopolist receives a prepayment equal to p_0 from every consumer type in $[T, 1]$ regardless of whether or not he buys in period 2 after c becomes common knowledge
- (2) period-2 profits are equal to zero when the product is profitable since the period-2 price is set equal to marginal cost, and
- (3) sales do not take place when the product is not profitable,

the monopolist's expected profits are equal to

$$E\pi = p_0(1 - T) + \beta(1 - T) \int_{c(T)}^{p_1} (p_1 - c) dc + \beta \int_0^{c(T)} \int_{\frac{p_1 - c}{\gamma - 1}}^1 (p_1 - c) d\alpha dc - F \quad (6)$$

where $F > 0$ is the R&D cost.

The last but one term in the profit function captures the effect of renegeing on the monopolist's expected profits. The lower the realized value of c below $c(T)$, the larger the

number of the ex ante period-1 customers who will renege ex post and buy one unit in period 2. The actual value of c plays an important role in determining how the monopolist's sales and profits are allocated between periods 1 and 2.

The monopolist's problem is to maximize expected profits, subject to equation (4), by choosing the amount of prepayment (p_0), the period-1 price (p_1) and the marginal consumer (T):

$$\max_{p_0, p_1, T} E\pi \quad \text{s.t.:} \quad \frac{1}{2}(\gamma - 1)^2 T^2 = p_0.$$

By substituting the constraint for p_0 into the objective, the expected profits function becomes:

$$E\pi = \frac{1}{2}(\gamma - 1)^2 T^2 (1 - T) + \beta(1 - T) \int_{c(T)}^{p_1} (p_1 - c) dc + \beta \int_0^{\frac{c(T)}{\gamma - 1}} (p_1 - c) dc - F.$$

By integrating and then using equation (3), we get the following expression for expected profits:

$$\begin{aligned} E\pi = & \frac{1}{2}(\gamma - 1)^2 T^2 (1 - T) + \frac{1}{2}\beta(\gamma - 1)^2 T^2 (1 - T) - \frac{1}{2}\beta(\gamma - 1)^2 T^2 \\ & + \frac{1}{2}\beta p_1^2 + \frac{1}{3}\beta(\gamma - 1)^2 T^3 - \frac{1}{3}\frac{\beta}{\gamma - 1} p_1^3 - F. \end{aligned} \quad (7)$$

The first-order conditions for expected profits maximization evaluated at the optimal solution (p_1^*, T^*) imply the following:

$$\frac{\partial E\pi(p_1^*, T^*)}{\partial p_1^*} = 0 \Rightarrow p_1^* = \gamma - 1 \quad (8)$$

$$\frac{\partial E\pi(p_1^*, T^*)}{\partial T^*} = 0 \Rightarrow T^* = \frac{2}{3 + \beta} \quad (9)$$

Using (9) and (4), we get the following expression for the expected profit-maximizing prepayment:

$$p_0^* = \frac{1}{2} p_1^{*2} T^{*2} = 2 \frac{(\gamma - 1)^2}{(3 + \beta)^2} \quad (10)$$

Finally, (7), (8) and (9) yield the following expression for the maximal expected profits:

$$E\pi^* = \frac{\beta^4 + 9\beta^3 + 27\beta^2 + 31\beta + 12}{6(3 + \beta)^3} (\gamma - 1)^2 - F \quad (11)$$

The previous analysis applies for the case where $T^* \geq p_1^*$ which implies that $\gamma \leq (5 + \beta)/(3 + \beta)$, using equations (8) and (9).

The following proposition summarizes the results which are illustrated in figure 3.

Proposition 1. Suppose that $\gamma \leq (5 + \beta)/(3 + \beta)$ and F is such that $E\pi^* > 0$. Then the expected profit-maximizing period-0 contract $\{p_0^*, p_1^*, p_1^*\}$ is such that

$$p_0^* = \frac{2(\gamma - 1)^2}{(3 + \beta)^2}$$

$$p_1^* = \gamma - 1$$

and p_2^* is equal to the realized value of the marginal cost.

Combining equations (8), (9) and (10), we get the following relation between p_0^* , p_1^* and T^* :

$$p_0^* = \frac{T^* p_1^{*2}}{3 + \beta}.$$

This equation reveals that there is a positive relation between the expected profit-maximizing prepayment (p_0^*) and marginal consumer T^* as well as between the expected profit-maximizing prepayment and period-1 price p_1^* . Note also that

- (1) T^* depends only on β
- (2) p_1^* depends only on γ , and
- (3) p_0^* depends on both β and γ .

We can explain the positive relation between p_0^* and T^* by exploiting their common dependence on β while the positive relation between p_0^* and p_1^* can be explained by using their common dependence on γ . We start with the former.

Proposition 1 and equation (9) reveal that p_0^* and T^* are inversely related to the monopolist's discount factor, β . A lower discount factor (given that γ is constant) induces the monopolist to raise the expected profit-maximizing prepayment since he now places a larger weight to getting more revenue in period 0 rather than later. A higher prepayment increases the marginal consumer type and decreases the number of consumers who choose to prepay it (top diagram in figure 2).

While p_0^* and T^* are inversely related to β , p_0^* and p_1^* are directly related to γ . Let us look at figure 4 and consider first what happens to p_0^* when γ increases (given that β is constant). A higher γ means that getting the product sooner rather than later becomes more important so that the number of consumers willing to pay the prepayment and purchase one unit of the product in period 1 increases. This effect is illustrated by a pivoting (around the origin) of $S(\gamma, t)$ to the left and the subsequent decrease of T^* to T^{**} , given the prepayment p_0^* . Nevertheless, equation (9) shows that the expected profit-maximizing marginal consumer type T^* is independent of the value of γ and varies only with β . As a result, the decrease in the value of the marginal consumer by $T^* - T^{**}$ should be outweighed by an equal-value increase. This can be achieved by increasing the prepayment since a higher prepayment implies that fewer consumers choose to prepay. Increasing the prepayment implies, therefore, that the marginal consumer increases. Furthermore, the prepayment should be increased by $p_0^* - p_0^{**}$ so that the net effect of the initial change in γ on T^* is nil.

In addition to the effects on p_0^* , a higher γ also increases the period-1 price (p_1^*) according to equation (8). This increase does not cause any further changes in figure 4 since the fact that $p_1^* = \gamma \cdot 1$ implies that

$$S(\gamma, t) = \frac{1}{2}(\gamma - 1)^2 t^2 = \frac{1}{2} p_1^{*2} t^2.$$

In other words, the effects of the increase in p_1^* on the function S have been already considered through the effects of the change in γ .

The increase in the period-1 price caused by the increase in γ is intuitive since a higher willingness to pay for immediate purchase of the product increases the monopolist's degrees of freedom to raise p_1^* . The increase in p_1^* implies a higher probability that the product will

be profitable since it becomes more likely that the realized cost value will be less than or equal to the increased period-1 price. As a result, the $1 - T^*$ consumer types are happy to pay a higher period-1 price because of their higher willingness to pay for immediate purchase of the good and they are also happy to pay a higher prepayment because the increase in the probability that the good will be profitable to produce is equivalent to an alleviation of the risk involved in paying the prepayment *and* not getting the product.

5. Welfare Implications of the Expected Profit-Maximizing Pricing Policy

In this section we will attempt to compare the expected welfare evaluated at the expected profit-maximizing solution of our model with the expected welfare that results from two alternative situations. We will refer to one of these as the "first-best" situation and to the other as the "no-prepayments" situation.

Under the first-best situation, no prepayments exist and all sales take place in period 1 at a price which is set equal to the actual value of the marginal cost which is realized in the beginning of period 1. In other words, the period-1 price is chosen *after* the actual value of the unit cost has become common knowledge. In the case of the no-prepayments monopolistic regime, the monopolist determines his profit-maximizing pricing policy (which does not involve prepayments) *after* the value of c becomes known in the beginning of period 1. Sales take place in *both* periods 1 and 2 so that the market segmentation which characterizes the model with prepayments (but not the first-best one) is carried through to this case as well.

The no-prepayments environment can be thought as an intermediate situation with the first-best and the prepayments models being the extremes. Moving from the first-best to the no-prepayments situation enables us to assess the effects (on expected welfare) of the market

segmentation based on the customers' time preference. Then, moving further to the situation which involves prepayments will help us assess the additional effects of introducing prepayments and determining the profit-maximizing pricing policy before the unit cost becomes known.

In section 5.1 we start with calculating the expected welfare as a result of the expected profit-maximizing pricing policy that involves prepayments and, then, we proceed with the expected welfare in the first-best and the no-prepayments environments (sections 5.2 and 5.3 respectively).

5.1. Expected Welfare When the Monopolist's Pricing Policy Involves Prepayments

Since expected welfare is given by expected consumer surplus plus expected profits and we already have an expression for expected profits [equation (1)], our task is to find an expression for expected consumer surplus. In order to do that, we will first compute the ex post consumer surplus for every possible value of the realized cost and then we will integrate ex post consumer surplus over all possible cost values.

Let us suppose that c_R describes the realized cost value. Then, we should distinguish among the following three cases and calculate ex post consumer surplus for each one of them separately.

5.1.1. Case 1: $c(T^) < c_R \leq p_1^*$*

Ex post consumer surplus is given by:

$$CS_1^* = \int_0^1 (\gamma t - p_0^* - p_1^*) dt + \int_{c_R}^{T^*} (t - c_R) dt$$

where p_1^* and T^* are given by equations (8) and (9) respectively.

The first term describes the surplus of consumers who buy in the first period whereas the second term is the surplus of consumers who buy in the second period. Consumer types who buy in the second period are identified as those for whom the utility derived from consuming one unit of the good (t) exceeds the price that they have to pay for this unit (c_R).

The final expression for CS_1^* is the following:

$$CS_1^* = \frac{-4\gamma^2 + 26 + 5\gamma - 4\beta\gamma^2 + 30\beta + \beta\gamma - 5\beta^2\gamma - \beta^3\gamma + 14\beta^2 + 2\beta^3}{2(3+\beta)^3} + \frac{1}{2}c_R^2 - \frac{2}{3+\beta}c_R$$

5.1.2. Case 2: $0 \leq c_R \leq c(T^*)$

This is the case where some consumers who have already prepaid have the opportunity to renege and buy in period 2 instead of period 1. All types in the interval $[T^*, (p_1^* - c_R)/(\gamma - 1)]$ will buy in period 2 (instead of period 1 that they were intending to buy in), where $(p_1^* - c)/(\gamma - 1)$ is the ex post marginal consumer type, since for these types the following is true:

$$\gamma t - p_1^* = t - c(t) < t - c_R$$

In other words, the ex post benefits from buying in period 2 are greater than those from purchasing the product in period 1.

Taking into account the existence of renegeing possibilities, ex post consumer surplus in this

case is given by

$$\begin{aligned}
 CS_2^* &= - \int_{\frac{p_1^* - c_n}{\gamma-1}}^1 p_0^* dt + \int_{\frac{p_1^* - c_n}{\gamma-1}}^1 (\gamma t - p_1^*) dt + \int_{c_n}^{\frac{p_1^* - c_n}{\gamma-1}} (t - c_n) dt \\
 &= - \frac{2(\gamma-1)^2(1+\beta)}{(3+\beta)^3} + \frac{1}{2} \gamma - \frac{1}{2} \frac{\gamma}{(\gamma-1)^2} (p_1^* - c_n)^2 - p_1^* + \frac{1}{\gamma-1} p_1^* (p_1^* - c_n) \\
 &\quad + \frac{1}{2(\gamma-1)^2} (p_1^* - c_n)^2 + \frac{1}{2} c_n^2 - \frac{1}{\gamma-1} p_1^* c_n + \frac{1}{\gamma-1} c_n^2
 \end{aligned}$$

5.1.3. Case 3: $p_1^* < c_n \leq 1$

If this is true, the monopolist does not produce the good and those consumers who have prepaid lose their prepayments. As a result, ex post consumer surplus is equal to the following

$$CS_3^* = - \int_{\tau^*}^1 p_0^* dt = - p_0^* (1 - \tau^*) = - \frac{2(\gamma-1)^2(1+\beta)}{(3+\beta)^3}$$

Given the above expressions for ex post consumer surplus, we can compute expected consumer surplus by integrating ex post surplus over all possible cost values:

$$ECS^* = \int_{c(\tau^*)}^{p_1^*} CS_1^* dc_n + \int_0^{\tau^*} CS_2^* dc_n + \int_{p_1^*}^1 CS_3^* dc_n$$

where $c(T^*) = (\gamma - 1)(1 + \beta)/(3 + \beta)$ using (3), (8) and (9).

Tedious algebra yields the following:

$$ECS^* = \frac{81\gamma^3 - 465\gamma^2 + 930\gamma - 72\beta^3 - 336\beta^2 - 716\beta + 12\beta^2\gamma^3 - 60\beta^3\gamma^2 - 282\beta^2\gamma^2 - 54\beta^2\gamma^3}{6(3 + \beta)^4} \\ + \frac{108\beta\gamma^3 - 564\beta^2\gamma + 120\beta^3\gamma - 596\beta\gamma^2 + 1192\beta\gamma - 546 - 5\beta^4\gamma^2 + 10\beta^4\gamma - 6\beta^4 + \beta^4\gamma^3}{6(3 + \beta)^4}$$

Expected welfare is found by adding ECS^* and $E\pi^*$ [equation (11)]:

$$EW^* = ECS^* + E\pi^* \\ = \frac{81\gamma^3 - 429\gamma^2 + 858\gamma - 18\beta^3 - 224\beta^2 - 611\beta + 12\beta^2\gamma^3 - 6\beta^3\gamma^2 - 170\beta^2\gamma^2 + 54\beta^2\gamma^3 - 108\beta\gamma^3}{6(3 + \beta)^4} \\ + \frac{340\beta^2\gamma + 12\beta^3\gamma - 491\beta\gamma^2 + 982\beta\gamma - 510 - 7\beta^4\gamma^2 - 14\beta^4\gamma + 6\beta^4 + \beta^4\gamma^3 - \beta^3\gamma^2 - \beta^3 - 2\beta^3\gamma}{6(3 + \beta)^4} - F$$

5.2. Expected Welfare in a First-Best Environment, EW^{FB}

In a first-best situation, one would expect that all sales would occur only in the first period at a price of $p_1 = c_R$. Furthermore, no prepayments would exist. Given these facts, a consumer of type t , $t \in [0, 1]$, will purchase one unit of the product if and only if $\gamma t \geq c_R$ or $t \geq c_R/\gamma$. This implies that our marginal consumer, T^{FB} , is equal to c_R/γ so that all consumer types in the interval $[c_R/\gamma, 1]$ are willing to purchase one unit of the product.

Since the seller makes zero profits from period-1 sales and all consumer types in $[c_R/\gamma, 1]$ purchase the product, ex post welfare is given by the following expression:

$$\begin{aligned}
 W^{FB} &= \int_{c_R/\gamma}^1 (\gamma t - c_R) dt - F \\
 &= \frac{1}{2} \gamma + \frac{1}{2} \frac{c_R^2}{\gamma} - c_R - F.
 \end{aligned}$$

The monopolist's discount factor does not appear in the above expression since period-1 profits are equal to zero.

Note that, since $c_R \in [0,1]$ and $\gamma > 1$, there always exists at least one consumer type t such that $\gamma t \geq c_R$. This implies that the product should be always produced from a social point of view. As long as the R&D cost (F) is sufficiently small so that ex post welfare (W^{FB}) is positive, the first-best environment guarantees the production of the good at a price equal to marginal cost. This is to be contrasted with the prepayment case where the product may not be produced even though there exists at least one type who values the good at an amount which is larger than its marginal cost of production.

Expected welfare in the first-best environment is given by the following:

$$EW^{FB} = \int_0^1 W^{FB} dc_R = \frac{\gamma}{2} + \frac{1}{6\gamma} - \frac{1}{2} - F.$$

5.3. Expected Profits and Welfare in the No-Prepayments Monopolistic Setting

In this kind of environment, we assume that (i) sales can take place in both periods 1 and 2, (ii) the monopolist chooses the period-1 price, p_1 , only after c_R has become known, and (iii) prepayments do not exist. Then, given p_1 and c_R each consumer type has to decide when to

buy.

The realized cost value, c_R , divides the set of consumer types into two subsets (shown in figure 5) and each type's decision as to the time of purchase depends on which of these subsets he belongs to.

Consider first the decision problem that each type t in $(c_R, 1]$ faces. Each such type derives positive net benefits from purchasing and consuming the good in period 2 since his gross benefits (t) are greater than the period-2 price (c_R). Furthermore, the marginal consumer type (denoted by T^1), i.e., that consumer type which is indifferent between buying in period 1 and period 2, is characterized by the following equation:

$$\gamma T^1 - p_1 = T^1 - c_R$$

Hence, the marginal consumer type is equal to

$$T^1 = \frac{p_1 - c_R}{\gamma - 1}. \quad (12)$$

As a result, whenever $t \in (c_R, 1]$, all consumer types in $[T^1, 1]$ will purchase the product in period 1.

Let us now consider the decision problem of each type t in $[0, c_R]$. No such consumer type would purchase the product in period 2 since if he did, he would derive non-positive net benefits. Consumer types in $[0, c_R]$, therefore, will only consider purchasing the product in period 1. All consumer types in $[T^2, c_R]$ will purchase the product in period 1, where T^2 is the marginal consumer type defined by the following equation:

$$\gamma T^2 - p_1 = 0.$$

The marginal consumer type is, therefore, equal to

$$T^2 = \frac{p_1}{\gamma} \quad (13)$$

There is an important remark to be made. The total period-1 demand does not consist of both consumer types in $[T^1, 1]$ and those in $[T^2, c_R]$. Alternatively, it is *not* the sum of $1 - T^1$ and $c_R - T^2$. Rather, the monopolist's period-1 demand is either $1 - T^1$ or $1 - T^2$ implying that the monopolist faces a kinked period-1 demand curve (illustrated in figure 6). Which section of the demand curve applies depends on a comparison between the cost of producing an additional unit of the good (c_R) and the price that period-1 customers have to pay for one unit of the good (p_1/γ) - where this price has been discounted by the factor γ in order to take into account the fact that some customers put extra value on consuming the good sooner rather than later.

We will proceed with examining three cases resting on this comparison. For each one of them, we will calculate ex post profits and welfare.

5.3.1. Case 1: $c_R < p_1/\gamma$

5.3.1.1. Consumers' and Monopolist's Behaviour

Using (12) and (13), $c_R \leq p_1/\gamma$ implies that $T^1 > T^2 > c_R$. Since $1 - T^1 < 1 - T^2$, period-1 demand is equal to $1 - T^1$. Given that $p_1 = c_R$, ex-post profits are equal to:

$$\pi^1 = B(p_1 - c_R)(1 - T^1) - F$$

$$= \beta(p_i - c_A) \left(1 - \frac{p_i - c_A}{\gamma - 1} \right) - F. \quad (14)$$

Maximizing profits by choosing p_i yields the following first-order condition with respect to p_i :

$$\frac{d\pi^1}{dp_i} = 0 = 1 - 2 \frac{p_i - c_A}{\gamma - 1} = 0 = \frac{p_i - c_A}{\gamma - 1} = \frac{1}{2} = T^1 = \frac{1}{2}. \quad (15)$$

Furthermore, we can use equation (15) to solve with respect to p_i :

$$p_i = c_A + \frac{1}{2}(\gamma - 1). \quad (16)$$

Using (15) and (16), (14) yields the following expression for profits:

$$\pi^1 = \beta \frac{\gamma - 1}{4} - F. \quad (17)$$

Finally, the fact that this case applies for $p_i > c_A$ implies that $c_A < 1/2$, using (16).

5.3.1.2. Ex Post Welfare

Ex post welfare is given by consumer surplus plus profits:

$$W^1 = \int_{T^1}^1 (\gamma t - p_i) dt + \int_{c_A}^{T^1} (t - c_A) dt + \pi^1$$

where π^1 is given by (17). The first integral represents the ex post surplus of period-1 customers and the second is the surplus of those buying in the second period.

Using (15) and (16), ex post welfare is given by the following equation:

$$W^1 = \frac{3}{8} + \frac{\gamma}{8} + \frac{c_R^2}{2} - c_R + \pi^1. \quad (18)$$

5.3.2. Case 2: $c_R > p_1/\gamma$

5.3.2.1. Consumers' and Monopolist's Behaviour

Similarly to the previous analysis, $c_R > p_1/\gamma$ implies that $T^1 < T^2 < c_R$. As a result, period-1 demand is equal to $1 - T^1 = 1 - p_1/\gamma$ and ex post profits are given by the following equation:

$$\pi^2 = \beta(p_1 - c_R) \left(1 - \frac{p_1}{\gamma} \right) - F.$$

Maximizing profits yields the following solution for p_1 :

$$\frac{d\pi^2}{dp_1} = 0 \Rightarrow p_1 = \frac{c_R + \gamma}{2}. \quad (19)$$

Using (19) and given that $T^2 = p_1/\gamma$, we have that

$$T^2 = \frac{c_R + \gamma}{2\gamma}. \quad (20)$$

Finally, the monopolist's profits at the profit-maximizing solution are

$$\pi^2 = \beta \left(\frac{c_R + \gamma}{2} - c_R \right) \left(1 - \frac{c_R + \gamma}{2\gamma} \right) - F \\ - \frac{1}{4} \beta \gamma - \frac{1}{2} \beta c_R + \frac{\beta}{4\gamma} c_R^2 - F.$$

This case applies only for $c_R > p_1/\gamma$ which implies that $c_R > \gamma(2\gamma - 1)$, using (19).

5.3.2.2. *Ex Post Welfare*

Welfare in this case is equal to the following:

$$W^2 = \int_{T^2}^1 (\gamma t - p_1) dt + \pi^2$$

where p_1 and T^2 are given by (19) and (20) respectively. There is no consumer buying in period 2 since $T^2 < c_R$.

After the substitutions and algebra, ex post welfare is equal to

$$W^2 = \frac{\gamma}{8} + \frac{c_R^2}{8\gamma} - \frac{c_R}{4} + \pi^2. \quad (21)$$

5.3.3. *Case 3: $c_R = p_1/\gamma$*

5.3.3.1. *Consumers' and Monopolist's Behaviour*

Since $T^2 = p_1/\gamma$, this case implies that $T^1 = T^2 = c_R$ so that period-1 demand is equal to $1 - T^1 = 1 - T^2 = 1 - c_R$. Furthermore, this case corresponds to the kink of the period-1 demand curve and holds for any c_R in the interval $[1/2, \gamma/(2\gamma - 1)]$.

Given that $p_1 = \gamma c_R$, the monopolist's profits are as follows:

$$\begin{aligned}\pi^3 &= \beta(\gamma c_R - c_R)(1 - c_R) - F \\ &= \beta(\gamma - 1)(c_R - c_R^2) - F.\end{aligned}$$

5.3.3.2. Ex Post Welfare

Ex post welfare for this case is given by (22):

$$W^3 = \int_{T^1}^1 (\gamma t - p_1) dt + \pi^3 = \frac{\gamma c_R^2}{2} - \gamma c_R + \frac{\gamma}{2} + \pi^3. \quad (22)$$

5.3.4. Expected Profits and Expected Welfare

Integrating ex post profits over all possible cost values yields expected profits:

$$\begin{aligned}E\pi &= \int_0^{1/2} \pi^1 dc_R + \int_{1/2}^{\frac{\gamma}{2\gamma-1}} \pi^3 dc_R + \int_{\frac{\gamma}{2\gamma-1}}^1 \pi^2 dc_R - F \\ &= \beta \frac{48\gamma^2 - 132\gamma^3 + 140\gamma^4 - 73\gamma^5 + 19\gamma^6}{24\gamma(2\gamma-1)^3} - F.\end{aligned} \quad (23)$$

$$EW = \int_0^{1/2} W^1 dc_R + \int_{1/2}^{\frac{\gamma}{2\gamma-1}} W^2 dc_R + \int_{\frac{\gamma}{2\gamma-1}}^1 W^2 dc_R + E\pi.$$

$$EW = \frac{24\gamma^3 - 38\gamma^4 + 31\gamma^3 - 19\gamma^2 + 7\gamma - 1}{24\gamma(2\gamma - 1)^2} + \beta \frac{48\gamma^2 - 132\gamma^4 + 140\gamma^3 - 73\gamma^2 + 19\gamma - 2}{24\gamma(2\gamma - 1)^3} - F.$$

5.4. Expected Welfare Comparisons

As we would expect, numerical calculations showed that the following is true, for every β in $(0,1]$ and γ in $(1, (5 + \beta)/(3 + \beta))$:

$$EW^{FB} > EW > EW^*.$$

Hence, expected welfare in the first-best situation is strictly greater than expected welfare in both the prepayment and no-prepayments situations. Furthermore, it is also true that expected welfare in the no-prepayments situation is strictly greater than expected welfare in a situation with prepayments.

Intuitively, there is a deadweight loss involved in moving from the first-best to the no-prepayments situation. The presence of this loss is due to two factors. First, some consumers have to purchase the product one period later than the period they would purchase it under the first-best situation. This is welfare-decreasing since all consumers place extra weight on having the good sooner rather than later. Second, we will show below that the no-prepayments situation may not guarantee the production of the good although the good should be always produced from a social point of view.

When the monopolist introduces prepayments and determines his profit-maximizing pricing

policy before the value of the unit cost becomes common knowledge, there is still a positive probability of non-production. Nevertheless, the monopolist's pricing policy is characterized by an inflexibility: his pricing policy is determined before the value of the unit cost is realized and is independent of that value. In relation to the no-prepayments case, this inflexibility imposes additional efficiency losses since the product will not be produced for any cost value that is greater than the period-1 price. If the monopolist had adopted the no-prepayments environment, production could have taken place (for a sufficiently small F) since the monopolist could then profitably adjust the period-1 price to the value of the unit cost.

Even though prepayments reduce expected welfare relative to the no-prepayments situation, the monopolist may adopt a pricing policy involving prepayments since there exist values for the monopolist's discount factor such that expected profits with prepayments are greater than expected profits for the no-prepayments case.¹ Specifically, this is going to be the case whenever the monopolist's discount factor is sufficiently small. Intuitively, the monopolist's placing a large weight on getting revenue before the value of the unit cost is revealed yields high profits only if the monopolist can exploit this characteristic through the use of prepayments which are the only way to bring in the desired revenue in period 0. On the other hand, if the monopolist does not place a large weight on having money in period 0 rather than later then he prefers to adopt the no-prepayments environment and set the period-1 price after the value of cost is known in order to take advantage of the flexibility in optimally adjusting his pricing policy to the value of the cost realization.

It is also true that there exist values for the R&D cost (F) such that (together with a sufficiently small β) expected profits with prepayments are positive while expected profits under the no-prepayments case are negative. For example, if $\beta = 0.1$ and $\gamma = 1.5$ then (11) and (23) imply that $E\pi^* = 19.09 - F$ and $E\pi = 0.00107 - F$. Hence, for any F such that $F \leq$

(0.00107, 19.09), it is true that $E\pi^* > 0$ while $E\pi < 0$. This result may suggest that, although prepayments reduce expected welfare relative to the no-prepayments situation, their presence can be tolerated if they provide the monopolist with revenues which are large enough to guarantee the production of the good.

6. The Prepayment as a Facilitating Device to Secure a Line of Credit

For the prepayment story to be plausible, it is essential that the monopolist's ability or inability to borrow from financial markets be explained. What is required for our argument to go through is a theory that shows that a firm may be unable to borrow if its net asset value falls below some critical level. Gale and Hellwig (1986) provide this link.

They consider a one-period model in which a debtor has a potential project with a random payoff whose expectation exceeds the required capital investment. The debtor is assumed to have insufficient funds to finance the project himself. A bank that finances the remainder only observes the actual return on the project if it incurs some auditing cost. They show that the optimal debt contract has the debtor reimburse the bank some predetermined amount if he chooses not to default. If the debtor defaults, the bank audits and confiscates the entire net return.

If the net asset value of the firm is low, however, so that the bank must finance most of the project, then the probability of audit will be quite high (since the firm will choose to default for moderate realizations of the return on the project). But then if the audit cost is high, the bank may be unwilling to finance the project at an interest rate that is worthwhile to the firm. Thus, a low net asset value for the firm may deprive it of access to financial markets.

It is reasonable to argue that the existence of a potential market for the monopolist's new product increases his net asset value. Prepaying buyers are a proof that such a market exists so that the prepayment can help the monopolist to secure a line of credit in order to finance some portion of the R&D cost.

7. Conclusion

The monopolist's expected profit-maximizing pricing policy can be thought of as a form of price discrimination. There is a sense in which this pricing policy combines elements of both second- and third-degree price discrimination. The monopolist offers two choices (either prepay p_0 and get one unit of the product in period 1 at a price of p_1 or not prepay anything and purchase one unit of the good in period 2 at a price of $p_2 = c_R$) which prompt consumers to separate themselves into two groups according to the importance they attach to early delivery of the product. Then, the monopolist charges these groups two different prices according to their "time elasticity of demand". Period-1 customers accept to pay a higher price than period-2 customers who can wait without considerable delay costs.

Even though prepayments have been shown to constitute a welfare-reducing device to earn higher revenues, the fact that they yield sufficiently high revenues to render a desirable product profitable in situations where, otherwise, it would not be produced, may make their presence unavoidable.

Notes

1. Subtracting (23) from (11) we get an expression for $E\pi^* - E\pi$ which is a 6th-degree polynomial of γ for any given value of β . Let's denote the biggest root of this polynomial by $R(\beta)$. Then, numerical calculations have shown that $E\pi^* - E\pi > 0$ for every β in $(0, 0.306)$ and γ in $(R(\beta), (5 + \beta)/(3 + \beta))$, where it is always true that $1 < R(\beta)$ for the above values of β . On the other hand, $E\pi^* - E\pi \leq 0$ for every β in $[0.306, 1]$ and γ in $(1, (5 + \beta)/(3 + \beta))$, where it is true that $(5 + \beta)/(3 + \beta) < R(\beta)$ for the above values of the discount factor.

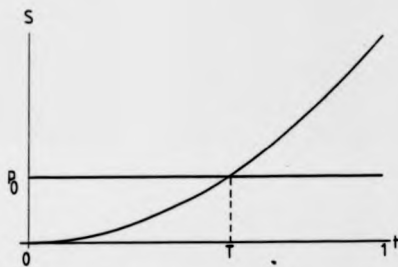


Figure 1. *The Marginal Consumer T Given the Period-0 Contract (p_0, p_1, p_2)*

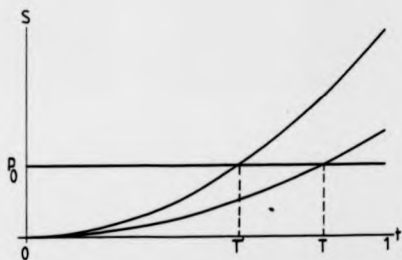
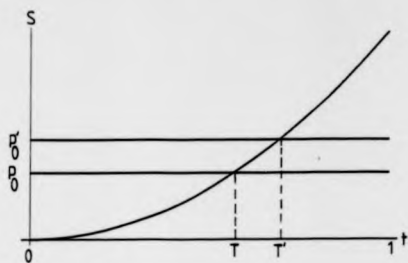


Figure 2. *Effects of Changes in p_0 and γ on the Marginal Consumer*

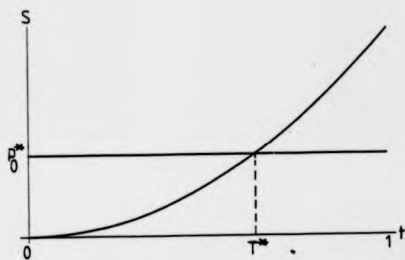


Figure 3. *The Expected Profit-Maximizing Solution*

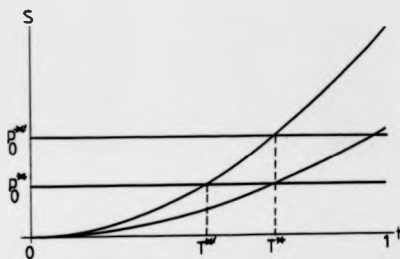


Figure 4. Effects of Changes in γ on T and p_0

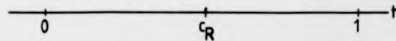


Figure 5. *Cost Realization and the Consumers' Decision in the No-Prepayments Monopolistic Setting*

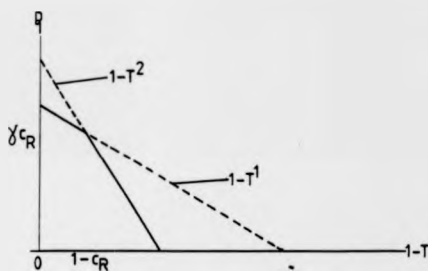


Figure 6. *The Period-1 Kinked Demand Curve When the Monopolist Does Not Use Prepayments*

1. Introduction

In this chapter we are interested in the role of *prepayments* as an integral part of the pricing strategy of an incumbent who operates in a market for a new product and is concerned with the protection of his monopoly power from the entry threats of a potential entrant. The incumbent may have been established as such by means of a successfully run R&D race. Nevertheless, the entrant may still threaten to enter with a very close substitute or by imitating the incumbent's product. We assume that this product is infinitely durable so that each consumer who buys it leaves the market forever. Furthermore, the number of potential customers in the beginning of the game is given and constant so that no new customers enter the market as the game proceeds.

The need for a pricing policy that will be effective in deterring entry emanates from our assumption that the incumbent is privately informed as to the level of cost. Thus, the entrant will consider entering the market only in the event that his cost is lower than the incumbent's cost.

It should be made clear from the outset that the focus of our analysis will be the game between the incumbent and the consumers rather than the entrant. The entrant will make his entry decision only after this game has reached an end and he will do so "passively". By this we mean that if consumers decide to purchase from the incumbent then the entrant will not

enter since there will be no market left. If, on the other hand, consumers do not purchase from the incumbent then the entrant will enter to serve the entire market. The incumbent, therefore, wants to convince consumers to purchase the product from him so that he can eliminate the threat of entry. In other words, the incumbent's behaviour aims at signalling *indirectly* (through the purchasing behaviour of the consumers) a low cost to the entrant.

Given the presence of cost uncertainty, consumers will not buy from the incumbent unless they know that his cost of production is lower than that of the entrant. If consumers know (or can infer) that the incumbent's cost is higher than the entrant's then they will not buy from the incumbent. If they did, they would miss the opportunity of purchasing the product from the entrant at a lower price which would be the result of the entrant undercutting the incumbent.

We will assume that the entrant's part in the play of the game between the incumbent and consumers, is confined to just observing the consumers' behaviour and acting accordingly. Thus, if consumers decide to buy from the incumbent then the entrant will conclude that the incumbent's cost is low enough to make entry unprofitable. If, on the other hand, consumers refuse to buy from the incumbent then this behaviour will be taken by the entrant to mean that the incumbent's cost is higher than his cost thus making entry profitable. In other words, Bertrand competition in prices implies that entry is profitable only if the incumbent's cost is higher than the entrant's. We will introduce a "small" positive cost of entry of ϵ in order to discourage the entrant from experimenting with entry in those situations where he infers that the incumbent's cost is lower than his. ϵ is assumed to be sufficiently small so that entry is not prevented whenever the entrant's cost is lower than the incumbent's cost. This assumption will not affect our results.

The above arguments establish the existence of the incentives of the low-cost incumbent

to reveal his type to consumers and of the high-cost incumbent to behave as if his cost were low. Hence, the low-cost incumbent will tend to choose a pricing policy that would be especially unattractive were the incumbent to have high costs. It then follows that the low-cost incumbent's choice will typically differ from the choice that he would make in an environment with complete information which, in turn, implies that there is a cost of asymmetric information due to signalling.

The next section overviews a particular example of a pricing policy that, if adopted by the low-cost incumbent, can reveal low cost to consumers.

2. Description of the Model

The main ingredient of the pricing policy we have in mind concerns the formation of a clientele with each consumer who is part of this clientele being willing to prepay for one unit of the product before production starts.

To make things clear, let us assume that there are three periods. In period 1, the incumbent (whose cost can be either higher or lower than the entrant's cost) spends an exogenously determined amount of money on the R&D of a new product whose production and sales take place in periods 2 and 3, respectively. The potential entrant makes his entry decision in the beginning of period 2 and should he decide to enter, he can enter only at that time. Bertrand competition in prices implies that entry is profitable only if the incumbent's cost is higher than the entrant's.

Suppose that in the beginning of period 1 the incumbent announces his pricing policy which involves the following elements. In period 1, the incumbent is prepared to sign a number of forward contracts requiring buyers who sign to prepurchase one unit of the

product. That is, buyers *prepay* for the product in period 1 and take delivery in period 3. The number of contracts to be signed is chosen by the incumbent and it can be less than the total number of consumers. Furthermore, contracts are signed on a first-come-first-served basis. Any consumers who do not have the chance to prepay will buy the product in period 3 by paying the monopoly price (which will be equal to the consumers' reservation price in the case of identical consumers).

After consumers get to know this pricing policy, they try to make an inference about the incumbent's true cost based on the size of the prepayment and the number of consumers who are allowed to prepay. The low-cost incumbent then will choose these variables in a way that would be unprofitable if the incumbent had high costs. In the separating equilibrium, therefore, consumers will buy from the low-cost incumbent only. Some of them will have the chance to prepay in period 1 for one unit of the product. The entrant then will not enter so that the remaining consumers will buy the product in period 3 by paying a monopoly price equal to their reservation price.

We will assume that if a separating equilibrium exists then the entrant will be able to infer whether or not the incumbent's cost is lower than his. If it is lower then the existence of some consumers who do not have the chance to prepay but, instead, wait until period 3 in order to purchase the product at the reservation price does not prompt the entrant to enter. Similarly, although the prepayment will be smaller than the reservation price, it may be higher than the entrant's cost. Even if this is true, the entrant will not enter against a more efficient incumbent. The above is reinforced by our assumption that there is an entry cost of $\epsilon > 0$.

The remainder of the paper is organized in the following sections. In section 3 we present the results of some relevant research. In section 4 we describe the ingredients of the model. In section 5 we examine the separating equilibria and in section 6 the "plausibility" of the

various separating equilibria is examined. The possibility of the incumbent participating in a pooling equilibrium is analyzed in section 7 and concluding remarks appear in section 8.

3. Review of Relevant Research

Although our model concentrates on the game between the incumbent and *consumers* (rather than the entrant) and thus entry is deterred *indirectly*, there is a relation with the limit pricing literature [for instance, Milgrom and Roberts (1982), Glazer and Israel (1990), Bagwell and Ramey (1988, 1991) and Srinivasan (1991)]¹ in the sense that limit pricing involves charging prices below the monopoly price to make new entry appear unattractive. In our case, the low-cost incumbent attempts to reveal his type and thus deter entry by inducing a sufficiently high number of consumers to prepay before production starts. The prepayment is smaller than the consumers' reservation price that the low-cost incumbent would charge in the absence of incomplete information and thus of any threat of entry.

There are two more points to be considered that differentiate our model from the ones mentioned previously. First, period-3 sales do not come from repeat purchases but from those consumers who did not prepay in period 1. Second, the incumbent chooses a prepayment and a *serving capacity* rather than a quantity. This implies that the period-1 prepayment and number of contracts are not related through a demand function in the sense that determination of the prepayment does not automatically imply a number of period-1 contracts. Both the period-1 prepayment and number of contracts have to be chosen by the low-cost incumbent in such a way that mimicking would be unprofitable if the incumbent had high costs. Hence, there are *two* signalling mechanisms in the model and the low-cost incumbent uses both: he signals his cost with the period-1 prepayment *and* number of contracts. This feature implies

a positive relation between the prepayment and the number of period-1 contracts: if the low-cost incumbent chooses to lower the cost-revealing prepayment then he can maintain the same level of profits *and* still prevent entry by also lowering the number of period-1 contracts.

Bagwell (1987) and Judd and Riordan (1989) have also considered games between consumers and an incumbent who is privately informed as to his level of cost.² Nevertheless, these models are different from ours in that the low-cost incumbent attempts to signal his type in order to attract repeat purchases (Bagwell) or induce consumers to draw more optimistic inferences about unknown quality from any particular price (Judd and Riordan). The effects of entry threats are not considered.

4. The Model

There are two firms (an incumbent and a potential entrant) and N buyers interacting for three periods in a market for a new product. Consumers are identical and have a reservation price of R with $R > 1$. They can buy one unit of the product either from the incumbent or the entrant if the latter decides to enter. The potential entrant has a known unit cost denoted by c_e , with c_e being a number in $(0,1)$, and faces a small cost of entry of $\epsilon > 0$. Although the buyers and the entrant do not directly observe the incumbent's unit cost, they do know that it is one of two possible levels, c_i and c_h , with $c_h \in (c_e, 1]$. For simplicity we assume that $c_i = 0$. Let $p^0 \in [0,1]$ be the buyers' prior probability assessment of the event that the incumbent's unit cost is $c_i = 0$.

To proceed, we formalize the market interaction between the firm and the potential customers as an extensive-form game having the following stages. First, "Nature" chooses the incumbent's cost, with p^0 being the probability that $c_i = 0$ is chosen. Next, the incumbent

observes c_i and chooses p and N_p ; $p \in (0, R]$ denotes the prepayment that is going to be paid in period 1 by $N_p \in (0, N]$ buyers for the purchase of the product which will be delivered in period 3. Buyers then observe p and N_p but not c_i , update beliefs to $p(p, N_p) \in [0, 1]$ and make their demand decisions. If $p(p, N_p) = 1$, all N consumers will purchase from the incumbent: N_p consumers will sign the first-period forward contracts on a first-come-first-served basis paying a prepayment price of p while the remaining $N - N_p$ will miss the opportunity to buy at the reduced prepayment price. Instead these customers will purchase the product in period 3 at a monopoly price equal to the reservation price R . The entrant, observing (p, N_p) and the behaviour of the buyers, decides not to enter. If, on the other hand, $p(p, N_p) = 0$, consumers do not purchase from the incumbent in which case the entrant enters in period 2 and undercuts the incumbent by charging a price of $c_h - \delta_I$. The entrant's profits are then equal to $(c_h - \delta_I - c_e - \varepsilon)N$.

We consider only pure-strategy sequential equilibria in this game. Let each buyer's strategy be denoted by $B(p, N_p) \in \{0, 1\}$, where $B = 1$ indicates that the consumer purchases from the incumbent. Furthermore, let $(p_i, N_{p,i})$, $i = I, h$, be the incumbent's choice of the prepayment and the number of contracts to be signed in the separating equilibrium when his true cost is c_i . Finally, $\pi(p, N_p, c_i, p(p, N_p))$ will denote the profits of the incumbent when his true cost is c_i and is perceived to be a low-cost type with probability $p(p, N_p)$.

In this setting, the collection $\{(p_i, N_{p,i}), (p_h, N_{p,h}), B(p, N_p), p(p, N_p)\}$ forms a sequential equilibrium if the following three conditions are satisfied [Overgaard (1991)].

Condition 1: Optimality For Incumbent. Let

$$S_p = \{p \in \mathbb{R}, | p \leq R\}$$

and

$$S_N = \{N_p \in I, | N_p \leq N\}.$$

Then, both types of the firm act in a sequentially rational manner; i.e., for $i \in \{I, H\}$

$$(p_i, N_{pi}) \in \underset{(p, N_p) \in S_p, S_H}{\operatorname{argmax}} \pi(p, N_p, c_i, p(p, N_p)).$$

Condition 2: Optimality for Buyers. For all (p, N_p) , $B(p, N_p) = 1$ if and only if $p(p, N_p) = 1$.

Condition 3: Consistency of Beliefs. Consumer beliefs are consistent [in the sense of Kreps and Wilson (1982)], i.e.,

- (a) if $(p_i, N_{pi}) \neq (p_h, N_{ph})$ then $p(p_i, N_{pi}) = 1$ and $p(p_h, N_{ph}) = 0$
- (b) if $(p_i, N_{pi}) = (p_h, N_{ph})$ then $p(p_i, N_{pi}) = p^0$ and
- (c) if $(p, N_p) \neq (p_i, N_{pi}), (p_h, N_{ph})$ then any $p(p, N_p) \in [0, 1]$ is consistent.

Condition 3(a) implies that in the separating equilibrium the low-cost incumbent must choose (p_i, N_{pi}) while the high-cost incumbent must choose (p_h, N_{ph}) . Whenever consumers observe (p_i, N_{pi}) then they know that the incumbent's cost is low and they update their beliefs to $p(p_i, N_{pi}) = 1$. If they observe (p_h, N_{ph}) , however, they infer that c_h is the true cost of the incumbent and their beliefs are updated to $p(p_h, N_{ph}) = 0$. In this case, the entrant enters and buyers purchase from the entrant.

If the incumbent chooses a pair (p, N_p) which is equal to neither (p_i, N_{pi}) nor (p_h, N_{ph}) , condition 3(c) implies that $p(p, N_p)$ can take any value. It is here that arbitrary off-the-equilibrium-path beliefs are allowed.

5. Sequential Separating Equilibria

In this model a separating equilibrium outcome consists of two prepayment-contract pairs (p_l, N_{pl}) and (p_h, N_{ph}) with $(p_l, N_{pl}) \neq (p_h, N_{ph})$, which enable consumers to infer with certainty the cost of the good supplied. In this section we begin with a characterization of the set of separating equilibria. A large set of possible (p, N_p) pairs will arise in separating equilibria. We will show that a reduced class of separating equilibria emerges when dominated strategies are eliminated.

To proceed with the determination of the set of sequential separating equilibria let us exploit the arbitrariness of off-equilibrium-path beliefs by setting $p(p, N_p) = 0$ for all $(p, N_p) \neq (p_l, N_{pl})$, i.e., high cost is inferred by the consumers whenever the expected low-cost equilibrium pair, (p_l, N_{pl}) , is not observed. This specification of beliefs is admissible because it obeys condition 3.

5.1. Sequential Rationality for the Low-Cost Incumbent

Given these beliefs then, the objective of the type l firm is to separate from type h by choosing a (p, N_p) pair that would be unattractive if the incumbent had high costs. Condition 1 of the definition of the equilibrium implies that sequential rationality is satisfied for the low-cost type when

$$\pi(p_l, N_{pl}, c_l, 1) \geq \max_{(p, N_p) \in S_p \times S_N} \pi(p, N_p, c_l, 0).$$

The right-hand side of this inequality describes the best that type l can get when $p(p, N_p) = 0$, i.e., when buyers believe that the incumbent has high costs. If buyers believe that this is the case, they refuse to purchase from him. The assumed "passive" behaviour of the entrant implies that the entrant will enter by "undercutting" the incumbent and charging a price of $c_a - \delta_l$. Nevertheless, the incumbent's true cost is $c_l = 0$ and as long as the entrant has already entered the market, Bertrand competition will drive the price down to $c_a - \delta_l = c_a$ with the low-cost type eventually being the only firm in the market. Given these considerations, the above inequality can be rewritten as follows.

$$p_l N_{pl} + R(N - N_{pl}) \geq c_a N \quad (1)$$

$$\Rightarrow p_l \geq R - (R - c_a) \frac{N}{N_{pl}} \quad (2)$$

Inequality (2) defines the set L' as

$$L' = \{(p_l, N_{pl}) \mid \pi(p_l, N_{pl}, c_l, 1) \geq c_a N\}.$$

Then, sequential rationality is satisfied for the low-cost type when

$$(p_l, N_{pl}) \in L'.$$

This set is illustrated in figure 1.

5.2. Sequential Rationality for the High-Cost Incumbent

Let us now consider the problem of the type- h incumbent. Sequential rationality is satisfied for the high-cost type when

$$\pi(p_l, N_H, c_h, 1) \leq 0$$

The left-hand side of the inequality is type h 's profits from mimicking the pricing strategy of the low-cost type. The right-hand side describes the maximal profits that the high-cost type gets assuming separation. These are zero (since consumers would purchase from the entrant), the same as the full-information level of profits.

The previous inequality can be rewritten as follows.

$$-(p_l - c_h)N_H + (R - c_h)(N - N_H) \leq 0 \quad (3)$$

$$\Leftrightarrow p_l \leq c_h + (R - c_h) \frac{N - N_H}{N_H}$$

Inequality (4) defines the set H^* as

$$H^* = \{(p_l, N_H) \mid \pi(p_l, N_H, c_h, 1) \leq 0\}.$$

We then conclude that sequential rationality is satisfied for the high-cost firm when

$$(p_l, N_H) \in H^*.$$

This is illustrated in figure 2.

5.3. Sequential Separating Equilibria

Combining the results of the two previous sections we obtain the following lemma which describes the set of prepayment-contract pairs that reveal low cost to consumers.

Lemma 1. *The set, S , of prepayment-contract strategies that support sequential separating equilibria is³*

$$\{(p_1, N_{\mu}) | (p_1, N_{\mu}) \in L^* \cap H^*\}.$$

Lemma 1 is illustrated in figure 3.

To establish existence of separating equilibria, we state the following theorem.

Theorem 1. *The set $L^* \cap H^*$ of sequential separating equilibria is always non-empty.*

Proof: See appendix A1.

The preceding arguments have shown that successful signalling by the low-cost type involves an appropriate choice of both the prepayment and the number of buyers who will prepay. Furthermore, the need for signalling will lead to a deviation of these variables from their complete-information levels. In the case of complete information the high-cost type would not produce while the low-cost type would make a profit of RN , i.e., no buyer would prepay. The existence of incomplete information, however, and the resulting need for signalling implies that the low-cost firm should try to appropriate some of the market in the first period by offering the chance to a *sufficiently high* number of buyers to buy the product

at a *sufficiently low* prepayment price. Choosing a low prepayment, p_l , without an appropriate choice of a value for N_{μ} cannot guarantee that the low-cost incumbent will be successful in signalling his type since, as is revealed from inequality (3), the high-cost incumbent could duplicate a strategy involving both a low p_l and a low N_{μ} . If that were the case, the high-cost type could make up for losses arising from selling to the N_{μ} buyers (at a prepayment price of p_l which is lower than c_h) through revenues from sales to the remaining market (at a price of R). As a result, a low prepayment must be accompanied by a high number of prepaying buyers if the high-cost incumbent is to make negative profits from duplicating the low-cost firm's pricing strategy. Nevertheless, at the same time p_l must not be too low and N_{μ} must not be too high if the incumbent is to find signalling profitable.

To conclude, for any pair $(p_l, N_{\mu}) \in L' \cap H'$, sequential rationality is satisfied for both types of the firm given the specified out-of-equilibrium beliefs. If $(p_l, N_{\mu}) \notin L' \cap H'$ then sequential rationality will fail to be satisfied for either type l or type h implying that such a pair cannot be part of a sequential equilibrium.

6. Undominated Sequential Separating Equilibria.

The previous section has demonstrated that, due to the arbitrariness of off-the-equilibrium-path beliefs, we get many separating equilibria. In this section we will argue that not all of the beliefs that support sequential equilibria are sensible.

Let us illustrate that non-sensible outcomes may be supported by strategies that are sequentially rational and beliefs that are in accordance with condition 3. Consider the pair (p_l^1, N_{μ}^1) in figure 3. This pair corresponds to an equilibrium only because buyers believe that a firm choosing (p_l^1, N_{μ}^1) is of a high cost, even though such a choice is dominated for a high-

cost type. This is because the best that can happen if a high-cost type chooses (p_i^i, N_{μ}^i) is that it is taken for a low-cost type, but this is worse than the worst that can happen, namely, that the firm is taken for a high-cost type [this is what inequality (3) describes]. If buyers believe that firms do not make dominated choices then $p(p_i^i, N_{\mu}^i)$ must be one, and the equilibrium where the low type chooses (p_i^i, N_{μ}^i) is overturned.

We will limit attention to equilibria that remain equilibria even after dominated strategies are removed from the game. Dominated strategies should never be played and beliefs should assign zero probability to such strategies. In particular, if a pair (p_i, N_{μ}) represents play of a dominated strategy for one type of firm but not for the other, beliefs following observation of such choice must ascribe zero weight to the type for which the strategy is dominated. We shall proceed, therefore, by refining the set of separating equilibria by eliminating dominated strategies, as proposed by Cho and Kreps (1987).

Their dominance criterion suggests that for out-of-equilibrium pair (p_i, N_{μ}) , firm type i may be eliminated for this pair if there is some other pair (p_i', N_{μ}') with

$$\min_{B \in (0,1)} \pi(p_i', N_{\mu}', c_i, B(p_i', N_{\mu}')) \geq \max_{B \in (0,1)} \pi(p_i, N_{\mu}, c_i, B(p_i, N_{\mu})).$$

This inequality states that a prepayment-contract strategy is weakly dominated for a type i if the implied profits under the most favourable buyer inferences are less than or equal to the profits that can be attained under the least favourable inferences.

This criterion suggests that consumers must assign zero probability to a type for which (p_i, N_{μ}) is weakly dominated, as long as (p_i, N_{μ}) is not dominated for the other type. If (p_i, N_{μ}) is weakly dominated for both types, arbitrary beliefs, $p(p_i, N_{\mu}) \in [0,1]$, can be specified. An equilibrium will be called *undominated* when $p(p_i, N_{\mu}) = 1$ (0) if (p_i, N_{μ}) is weakly dominated

for type h (l) but not for type l (h).

Now we can apply the above to our context. From the construction of H^* , we know that for all $(p_i, N_{\mu}) \in H^*$

$$\pi(p_i, N_{\mu}, c_h, 1) \leq 0$$

and (p_i, N_{μ}) is weakly dominated for the high-cost firm, but not for the low-cost firm as long as $(p_i, N_{\mu}) \in \text{int}(L')$. Buyer beliefs, therefore, must assign $\rho(p_i, N_{\mu}) = 1$ to any $(p_i, N_{\mu}) \in L' \cap H^*$ where $\pi(p_i, N_{\mu}, c_h, 1) > c_l N$, and the low-cost type should maximize profits given these beliefs. This, in turn, implies that the low-cost firm can successfully signal its type by maximizing profits subject to the constraint that $(p_i, N_{\mu}) \in H^*$ since all such pairs are weakly dominated for the high-cost type. These considerations lead to the following lemma.

Lemma 2. *In any undominated sequential separating equilibrium, the low-cost type chooses (p_l^*, N_{μ}^*) such that*

$$(p_l^*, N_{\mu}^*) \in \underset{(p_i, N_{\mu}) \in H^*}{\text{argmax}} \pi(p_i, N_{\mu}, c_l, 1).$$

The next task is to characterize the set of (p_l^*, N_{μ}^*) pairs that obeys lemma 2. We will show that the problem of the low-cost type does not have a unique maximizer. We state the following theorem.

Theorem 2. *The set S^* , with $S^* \subset S$, of (p_l^*, N_{μ}^*) pairs which supports undominated sequential separating equilibria is the boundary of the set H^* and is given by*

$$\{(p_l^*, N_{\mu}^*) \mid \pi(p_l^*, N_{\mu}^*, c_l, 1) = c_h N\}.$$

Proof: See appendix A2.

Theorem 2 is illustrated in figure 4.

As is stated in the theorem, any pair (p_l^*, N_{μ}^*) such that $\pi(p_l^*, N_{\mu}^*, c_h, 1) = c_h N$ constitutes an undominated sequential separating equilibrium outcome. All these pairs lie on the boundary of H^* and they are equally desirable in the sense that they all successfully signal low cost and yield the same profit.

Intuitively, the reason that successful signalling requires that the low-cost type's profits be less than or equal to $c_h N$ is that if the high-cost type were to mimic the pricing policy of the low-cost type, his profits would be

$$\begin{aligned}\pi(p_l^*, N_{\mu}^*, c_h, 1) &= (p_l^* - c_h)N_{\mu}^* + (R - c_h)(N - N_{\mu}^*) \\ &= p_l^* N_{\mu}^* + R(N - N_{\mu}^*) - c_h N \\ &= \pi(p_l^*, N_{\mu}^*, c_l, 1) - c_h N.\end{aligned}$$

Hence, if the low-cost type's profits are less than or equal to $c_h N$ [$\pi(p_l^*, N_{\mu}^*, c_h, 1) \leq c_h N$] then the high-cost type's profits will be less than or equal to zero rendering mimicking unprofitable.

Note that it can be optimal for the low-cost incumbent to price discriminate over time. This can be seen from the fact that $N_{\mu}^* > 0$ for every p_l^* in $(0, c_h]$. In other words, the low-cost incumbent will always sign more than $(R - c_h)N/R > 0$ contracts thus rationing consumers over time.

The fact that N_{pl}^* is positive is required by the need to signal low cost to consumers. If N_{pl}^* were set equal to zero then the high-cost incumbent would make positive profits by mimicking the low-cost incumbent's pricing policy. Using (3), his profits would be

$$\pi(p_l^*, 0, c_h, 1) = (R - c_h)N$$

which is positive for every p_l^* in $(0, c_h]$. Hence, the low-cost incumbent must choose a number of contracts such that $N_{pl}^* > (R - c_h)N/R$.

Although signing a positive number of contracts in period 1 is inevitable, rationing consumers by price discriminating over time is not. If we assume that there is a small cost associated with price discrimination then the low-cost incumbent will prefer to sign a number of contracts equal to the total number of consumers. In this case, the pair (c_h, N) would become a focal point and every consumer would prepay in period 1.

7. Pooling Equilibria

Thus far we have considered equilibria in which buyers become fully informed, but there may also exist equilibria in which buyers learn nothing at all from observing the prepayment and the number of first-period contracts. These pooling equilibria are characterized by $(p_l, N_{pl}) = (p_h, N_{ph})$. This implies that buyers receive no information on cost from the equilibrium play which implies that $p(p_l, N_{pl}) = p^0$.

One reason why we want to examine pooling equilibria is that successful signalling by the low-cost firm involves a significant signalling cost. To attain the maximum profit of $c_h N$ in the separating equilibrium, the type l firm must choose $(p_l^*, N_{pl}^*) \in S^*$ with $p_l^* \in (0, c_h]$ and N_{pl}^*

$\in ((R - c_h)N/RN]$. This is different, however, from the full-information optimum in which all buyers pay their reservation price for one unit of the product and the low-cost incumbent gets a profit of RN . Hence, signalling implies that profits are lower than their full-information level by an amount equal to $(R - c_h)N > 0$.

Nevertheless, the low-cost incumbent would never agree to participate in a pooling equilibrium since a sufficiently small cost of entry guarantees that entry is favourable under the entrant's prior beliefs. To see this, consider the entrant's expected profits from entry:

$$E\pi_e = p^0 \pi_e^l + (1 - p^0) \pi_e^h - e$$

where π_e^l and π_e^h are the entrant's profits when he enters against a low-cost and a high-cost incumbent respectively. Since $\pi_e^l = 0$, $\pi_e^h = (c_h - c_l)N$ and e is sufficiently small, we have that $E\pi_e > 0$ so that there is always going to be entry. This, in turn, implies that the maximum average price that buyers are willing to pay is

$$p^0 c_e + (1 - p^0) c_h < c_h < R$$

This implies that the low-cost incumbent's profit will be lower than $c_e N$ which is the profit he would get in a separating equilibrium. As a result, the low-cost incumbent would never participate in a pooling equilibrium since separating yields larger profits and also prevents entry. Hence, if the "technology" of prepayments exists, the low-cost type will always choose to undertake a signalling behaviour. Non-selection of the prepayment as a signal implies that the incumbent's cost is high.

8. Concluding Remarks

We have presented a model in which a low-cost incumbent tries to convince consumers to prepurchase the commodity before its production actually starts, leaving no market for the entrant. One way to convince at least some consumers to prepay is to offer the commodity in both a forward and a spot market. For this pricing policy to convey any information about the incumbent's cost it must be the case that (a) the prepayment is lower than the high-cost type's cost and (b) the number of contracts is sufficiently high to prevent profitable mimicking by the high-cost type. Any prepayment and/or number of contracts that do not comply with the rules implied by (a) and (b) will induce entry and deprive the low-cost type of the benefits from both period-1 and period-3 purchases.

Although the low-cost incumbent may be successful in eliminating the threat of entry by choosing a pair in S^* , there are many such equilibrium pairs to choose from. In the next chapter, we will try to come up with a unique separating equilibrium by considering the case where consumers are heterogeneous in terms of their reservation price and the producer rations consumers between periods 1 and 3 according to two different rationing rules.

Appendix

A1. *Proof of Theorem 1.* To prove that the set $L' \cap H'$ is non-empty, it is sufficient to prove that

$$c_h - (R - c_h) \frac{N - N_{pl}}{N_{pl}} > R - (R - c_h) \frac{N}{N_{pl}}.$$

This inequality can be rewritten as follows.

$$\frac{R - (R - c_h) \frac{N}{N_{pl}}}{c_h} > \frac{N}{N_{pl}},$$

This is true by assumption. Hence, $L' \cap H'$ is non-empty.

A2. *Proof of Theorem 2.* The low-cost type's decision problem can be stated as follows.

$$\begin{array}{ll} \max & \pi(p_1, N_{pl}, c_1, 1) \\ \text{s.t.} & (p_1, N_{pl}) \in H^2 \end{array}$$

Substituting for the profit function and the constraints we get

$$\begin{array}{ll} \max_{(p_1, N_{pl}) \in S_p \times S_N} & [p_1 N_{pl} + R(N - N_{pl})] \\ \text{s.t.} & p_1 \leq c_h - (R - c_h) \frac{N - N_{pl}}{N_{pl}} \end{array}$$

The Lagrangian is defined as follows.

$$\Lambda = p_l N_{pl} + R(N - N_{pl}) + \lambda \left[c_h - (R - c_h) \frac{N - N_{pl}}{N_{pl}} - p_l \right].$$

The first-order conditions for a maximum are

$$\begin{aligned}\Lambda_{p_l} &= N_{pl} - \lambda = 0 \\ \Lambda_{N_{pl}} &= p_l - R + \lambda(R - c_h) \frac{N}{N_{pl}^2} = 0 \\ \Lambda_\lambda &= c_h - (R - c_h) \frac{N - N_{pl}}{N_{pl}} - p_l \geq 0 \quad \wedge \quad \lambda \Lambda_\lambda = 0.\end{aligned}$$

The problem has an interesting solution when $\lambda > 0$ in which case $N_{pl} = \lambda > 0$ and $\Lambda_\lambda = 0$.

Substituting N_{pl} for λ in the second first-order condition and solving with respect to p_l we get

$$p_l = R - (R - c_h) \frac{N}{N_{pl}}.$$

Solving the third first-order condition with respect to p_l we get exactly the same solution as above. Thus, we have one equation and two unknowns implying that all pairs (p_l^*, N_{pl}^*) such that

$$\begin{aligned}p_l^* &= R - (R - c_h) \frac{N}{N_{pl}^*} \\ &= c_h - (R - c_h) \frac{N - N_{pl}^*}{N_{pl}^*}\end{aligned}$$

are acceptable solutions and lie on the upper boundary of H' .

Notes

1. Bagwell and Ramey (1988, 1991) extend Milgrom and Roberts' (1982) model in two different ways. In their 1988 model, they allow an incumbent to signal his costs with both price and advertisements. They show that preentry price is distorted downward and demand-enhancing advertising is distorted upward, as a consequence of signalling. In their 1991 model, they allow for multiple incumbents. Each incumbent is informed as to the level of an industry cost parameter and selects a preentry price while a single entrant observes each incumbent's preentry price. The authors find that incumbents are unable to coordinate deception, which results in a separating equilibrium in which preentry prices are not distorted. Srinivasan (1991) investigates the strategic pricing behaviour of an incumbent who operates in multiple markets. It is shown that the low-cost incumbent minimizes the cost of communicating the true cost type to an uninformed potential entrant by combining the signalling effort across markets, instead of independent signalling in each market. It is also established that, in the combined signalling effort, the low-cost incumbent limit prices in each market. Glazer and Israel (1990) attempt to show that there exist signalling mechanisms which are less costly than the limit pricing one. They study an example in which the contract offered to the manager of a monopolistic firm may induce him to take some actions that will credibly signal the firm's marginal cost and will deter entry if the firm is sufficiently efficient.

2. Bagwell (1987) explores the hypothesis that a firm may have an introductory sale in order to signal that cost is low and, hence, that price will be low in the second period. In this way, an introductory sale can lead to repeat business and, possibly, gains in overall profits. When consumers must pay a search cost in order to learn a firm's current price, a low-cost producer

will have an introductory sale in the separating equilibrium. Judd and Riordan (1989) show that, in the absence of cost asymmetries across different qualities or repeat purchases, equilibrium prices are distorted to signal both the firm's costs as well as its quality information. Specifically, the introductory price for the product is distorted below its complete-information monopoly level in order to signal low cost, and subsequently, price is distorted above its full-information monopoly level in order to signal high quality.

3. For simplicity, we have omitted the prepayment-contract strategy pairs of the type-*A* firm since this type does not produce in a separating equilibrium of this model.

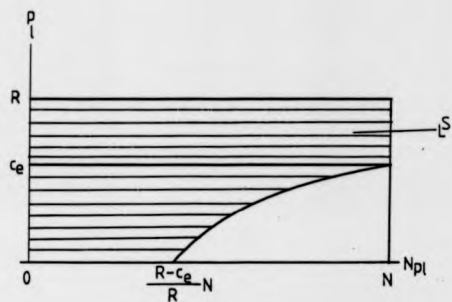


Figure 1. *Sequential Rationality for the Low-Cost Type*

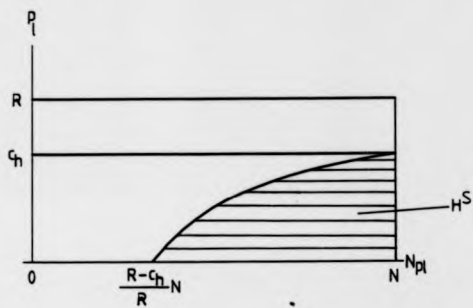


Figure 2. Sequential Rationality for the High-Cost Type

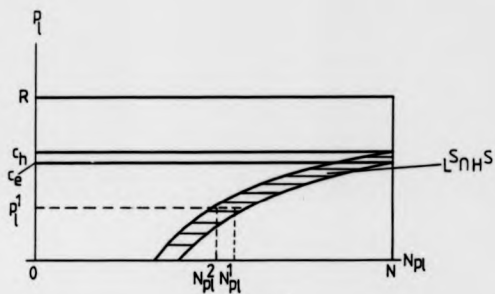


Figure 3. Sequential Separating Equilibria

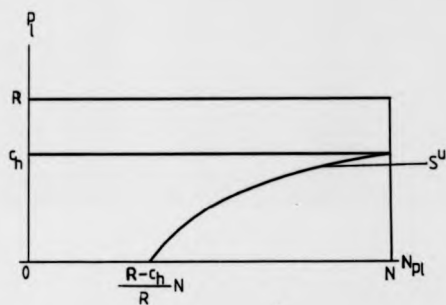


Figure 4. *Undominated Sequential Separating Equilibria*

5 PREPAYMENTS AND CONTRACTS AS SIGNALS OF COST IN A MODEL WITH HETEROGENEOUS BUYERS

1. Introduction and Summary of the Model

In chapter 4, we have analyzed a three-period model in which an incumbent seller is privately informed as to whether his costs are higher or lower than the entrant's costs. In period 1 the incumbent spends an exogenously determined amount of money on the R&D of a new product whose production and sales take place in periods 2 and 3, respectively. The potential entrant makes his entry decision in period 2 and should he decide to enter, he can enter only at that time incurring a small entry cost of ϵ . Bertrand competition in prices implies that entry is profitable only if the incumbent has high costs. In this model, the low-cost incumbent attempts to reveal his type (in order to convince consumers to purchase the product from him and forestall period-2 Bertrand competition which would lower his price down to the entrant's cost level) by building up a customer base before production starts and before the entrant makes his entry decision. Consumers will be happy to become part of this base only if they are convinced that the incumbent's costs are low. The reason for this is that if costs are low then entry will not take place and consumers who form that base will pay a prepayment price which will be lower than the reservation price that the remaining consumers will have to pay in period 3. The low-cost incumbent chooses the prepayment and the number of buyers who will prepay in order to signal his low cost to consumers (and the entrant).

In this chapter, we modify the above model by assuming that consumers are heterogeneous and indexed by their reservation price. We further assume that the incumbent has two choices as to the pattern of serving customers who stand willing to prepay. The introduction of these two assumptions will give rise to a *unique* separating equilibrium outcome (thus avoiding the multiplicity problem which is present with homogeneous buyers even after the elimination of dominated strategies) and will add a qualification to the result of chapter 4. Specifically, *rationing* of consumers in the equilibrium can be optimal from the point of view of the low-cost incumbent. In the context of this chapter, we will use the term "rationing" to describe a situation where it is optimal for the low-cost incumbent to satisfy only a part of the demand for period-1 contracts involving prepayments. If rationing actually takes place in the separating equilibrium, rationed consumers will be able to purchase the product from the low-cost incumbent in period 3 by paying a monopoly price which will be higher than the prepayment. Whether or not rationing will emerge in equilibrium depends on the "serving scheme" which defines the types of consumers that are allowed to prepay.

In this model, we will use the terms "serving pattern" or "serving scheme" to describe different ways of serving customers. These patterns are chosen by the incumbent from the outset and they are designed to create a period-1 customer base composed of some particular types of consumers. The term "rationing" will be reserved to characterize the equilibrium outcome. Hence, while the serving scheme is chosen by the incumbent before the start of the game, rationing is an endogenous feature and its presence will depend on the kind of the serving pattern that has been chosen by the incumbent. We will consider two serving schemes. The *efficient-serving* scheme and the *proportional-serving* scheme.¹

Under the efficient-serving scheme, only consumers with the highest reservation prices are allowed to prepay. It will be shown that rationing will not be optimal from the point of view

of the low-cost firm. Hence, everyone who will be willing to prepay will be allowed to do so. This implies that all consumers served by the low-cost firm pay the same prepayment price (which will be equal to the period-3 monopoly price) and are delivered the product after it is produced. In this case, the low-cost incumbent signals his cost and thus prevents entry by building up a customer base which is formed by the highest-reservation price buyers while he gives zero weight to monopoly profits. This is to be contrasted with the case of homogeneous buyers where we obtained a set of separating equilibria rather than a unique one. That set contained a non-rationing equilibrium in which every consumer prepaid the (common) reservation price R .

In the case of the proportional-serving scheme, all consumers have the same probability of prepaying and rationing becomes optimal for the low-cost incumbent who prefers to serve only a subset of the set of consumers who are willing to prepay. Buyers who do not have the chance to prepay, purchase the product by paying a higher monopoly price in period 3. This is both similar to the case of homogeneous buyers in that buyers who do not prepay have to pay a higher price later and, at the same time, different from that case in that there is a *unique* equilibrium outcome involving rationing.

The remainder of the paper is organized in the following sections. In section 2 we describe some previous work on rationing and signalling. A description of the main elements of the model is presented in section 3. In section 4 we describe the efficient-serving scheme and present the results that this scheme gives rise to. The proportional-serving scheme is analyzed in section 5. Section 6 examines some comparative statics. Concluding thoughts appear in section 7.

2. Review of Previous Research

Equilibria with rationing have been investigated by a number of authors for a wide range of contexts. For example, Kreps and Scheinkman (1983) analyze a two-stage model of oligopoly in which firms choose their capacities before engaging in Bertrand-like price competition. They demonstrate that, under certain assumptions about how demand is rationed when the lower-priced firm cannot meet market demand, the unique perfect equilibrium outcome coincides with the Cournot outcome. Davidson and Deneckere (1986) show that the result of Kreps and Scheinkman is sensitive to the specification of residual demand. They suggest an alternative rationing rule for which the Cournot outcome cannot emerge in equilibrium. Wilson (1988) provides an explanation of price dispersion by a monopolist, assuming that consumers arrive in a random order and are served on a first-come-first-served basis. A firm can sometimes increase its profits by charging two different prices for the same item and rationing sales at the lower price. Boyer and Moreaux (1988, 1989) characterize the equilibria of Stackelberg duopolies with differentiated products, where the firms are fighting either in prices and quantities or in prices and serving capacities. It is shown that, in the price-quantity case, there is always rationing by the leader. In the price-serving capacity case, the leader is rationing only if the goods are close substitutes. Carlton (1991) presents a more general theory which implies that rationing is to be expected whenever the assumption of costless use of the price system is relaxed.

Nevertheless, what is closely related to our work is the use of the combination of *rationing and underpricing* for the purpose of achieving signalling aims. This combination has been used by Allen and Faulhaber (1989) for the case of the initial-public-offerings (IPOs) market. They develop a theory consistent with some empirical evidence suggesting the existence of "hot-issue" markets for IPOs: in certain periods and in particular industries, issues are underpriced and rationing occurs. This observation can be explained by a model which

assumes that the firm itself has the best knowledge of the success prospects of implementing an innovation. Under certain circumstances, firms with the most favourable prospects find it optimal to signal their type by underpricing their initial issue of shares. Investors know that only the firms with the highest success probabilities can recoup the cost of this signal from subsequent issues. It is shown then, that underpricing can signal favourable prospects for the firm and that it occurs in a separating equilibrium.

3. Main Elements of the Model

As in chapter 4, an incumbent, a potential entrant and the buyers interact for three periods in a market for an homogeneous good. In the initial period the incumbent spends some money on the development of a new product. At the start of the second period where production takes place, the entrant may choose to enter incurring a small cost of entry of $e > 0$. The presence of this cost serves the same purpose as in chapter 4. Sales take place in period 3. Potential customers and the entrant make their purchase and entry decisions without having complete information about the incumbent's production costs, though they might be able to infer cost information by observing the incumbent's first-period prepayment-contract decision.

We assume that there is a continuum of consumers indexed by their reservation price, R , which is described by a uniform distribution with support on the interval $[0,1]$. Consumers purchase one unit of the product either from the incumbent or the entrant if the latter decides to enter. The potential entrant's unit cost is denoted by c_e , with c_e being a known number in $(0,1)$. Although the buyers and the entrant do not directly observe the incumbent's unit cost, c_i , they do know that it is one of two possible levels, c_1 and c_2 with $0 \leq c_1 < c_e < c_2 \leq 1$. For computational convenience we are going to assume that $c_1 = 0$. Let $\rho^0 \in [0,1]$ be the buyers'

prior probability assessment of the event that the incumbent's unit cost is c_l .

We formalize the market interaction between the firm and the potential customers as an extensive-form game having the following stages. First, "Nature" chooses the incumbent's cost, with p^0 being the probability that $c_l = c_l$ is chosen. Next, the low-cost incumbent observes c_l and chooses his pricing policy with the purpose of revealing his type and thus attracting consumers and preventing entry. His pricing strategy is characterized by the following elements. The incumbent is prepared to sign contracts requiring consumers who agree to sign these contracts to prepay for one unit of the product in the first period and wait for delivery until the third period. Both the prepayment and the number of contracts to be signed are chosen by the low-cost type in such a way that mimicking is unprofitable from the point of view of the high-cost type. Buyers then observe the prepayment and the number of contracts to be signed (but not c_l), update their beliefs to $p \in [0,1]$ and make their purchase decisions. If $p = 1$, potential customers decide to purchase from the incumbent. This implies that the entrant will not enter. Production will take place in period 2 and delivery in period 3. The size of the prepayment and the number of period-1 contracts depend on the serving scheme chosen by the incumbent or, in other words, on the consumer types that are allowed to prepay. Two serving schemes are considered in the rest of the paper.

4. The Efficient-Serving Scheme

4.1. Description of the Efficient-Serving Scheme

Let us start the analysis by assuming that p_1 is the prepayment set by the low-cost incumbent in the separating equilibrium. Then, consumers with reservation prices greater than or equal

to p_i are willing to prepay.

This set of consumer types is described by

$$N_i = \int_{p_i}^1 dR - 1 - p_i$$

The relation between p_i and N_i is illustrated graphically in figure 1.²

Nevertheless, it may be optimal for the low-cost type to sign a limited number of contracts which allows only N_{pi} consumers to prepay. These customers will be the ones with the highest reservation prices:

$$N_{pi} = \int_Q^1 dR - 1 - Q < N_i$$

Whenever N_{pi} is strictly less than N_i then the efficient-serving scheme implies a residual demand (QA in figure 1) for the low-cost firm. Since the pair (p_i, N_{pi}) reveals the low-cost type's cost and thus prevents entry, the low-cost firm behaves as a monopolist towards the residual demand, charging a price of p_{mi} . Hence, given a monopolistic price of p_{mi} , the residual demand N_{ri} consists of consumers with reservation prices $R \in [p_{mi}, Q]$:

$$N_{ri} = \int_{p_{mi}}^Q dR - Q - p_{mi} - 1 - N_{pi} - p_{mi}$$

If rationing turns out to be an optimal strategy in the separating equilibrium, it will be true that $N_{pi} < N_i$ and $p_{mi} > p_i$. In other words, buyers who sign contracts obtain a discount as

opposed to those who do not sign contracts and pay a higher monopoly price.

4.2. Sequential Separating Equilibria

We will consider only pure-strategy equilibria. Let each buyer's strategy be denoted by $B(p, N_p) \in \{0, 1\}$, where $B = 1$ indicates that the consumer buys from the incumbent. In our setting the collection $\{(p_l, N_{pl}), (p_h, N_{ph}), B(p, N_p), \rho(p, N_p)\}$ forms a sequential equilibrium if three conditions are satisfied. These have already been mentioned in chapter 4 and there is no need to repeat them here.¹

Before proceeding with the determination of the unique separating equilibrium, let us exploit the arbitrariness of off-the-equilibrium-path beliefs by setting $\rho(p, N_p) = 0$ for every $(p, N_p) \neq (p_l, N_{pl})$, i.e., high cost is inferred by consumers whenever the expected low-cost equilibrium pair (p_l, N_{pl}) is not observed. Given this specification, we can now apply condition 1 of the definition of an equilibrium.

4.2.1. Sequential Rationality for the Low-Cost Type

Given these beliefs then, the objective of the type l firm is to separate from type h by choosing a (p, N_p) pair that would be unattractive to choose if the incumbent had high costs. Condition 1 of the definition of an equilibrium implies that sequential rationality is satisfied for the low-cost type in the separating equilibrium when

$$\pi(p_l, N_{pl}, p_{ml}, N_{ml}, c_l, 1) \geq \max_{(p, N_p) \neq p_l, N_{pl}} \pi(p, N_p, c_l, 0)$$

where p_{ml} maximizes profits from sales to the residual demand and N_H describes the set of consumer types that pay p_{ml} .

The right-hand side of this inequality describes the best that the low-cost firm can get when $\rho(p, N_p) = 0$, i.e., when buyers believe that the incumbent has high costs. If buyers believe that the incumbent's type is c_h , they refuse to buy from him. This behaviour induces the entrant to enter by "undercutting" the incumbent and charging a price of $c_h - \delta_1$. Nevertheless, the incumbent's true cost is $c_l = 0$ and as long as the entrant has already entered the market, Bertrand competition will drive the price down to $c_h - \delta_2 = c_l$ with the low-cost type eventually being the only firm in the market. Then, we can write the previous inequality as follows.

$$p_l N_{pl} + p_{ml} N_H \geq c_h \int_{c_h}^1 dR$$

$$- p_l N_{pl} + p_{ml} (1 - N_{pl} - p_{ml}) \geq c_l (1 - c_h). \quad (1)$$

The first term in the left-hand side describes profits from prepayments. The second term is the maximal monopoly profits from sales to the residual demand:

$$p_{ml} (1 - N_{pl} - p_{ml}) = \max_p p (1 - N_{pl} - p)$$

where

$$p_{ml} = \frac{1 - N_{pl}}{2} - \frac{Q}{2}.$$

The fewer the consumers that prepay, the higher the residual demand and the higher is the period-3 monopoly price. The above equation suggests that the low-cost type is only indirectly committed to a period-3 monopoly price through the optimal choice of the number of contracts. This is to be contrasted to the proportional-serving scheme where, as we shall see, the low-cost firm is committed from the outset to a constant period-3 monopoly price which is independent of the choice of N_{pl} .

By substituting for p_{mh} , inequality (1) becomes as follows.

$$\begin{aligned} p_l N_{pl} + \frac{(1 - N_{pl})^2}{4} &\geq c_s(1 - c_s) \\ \Leftrightarrow p_l &\geq \frac{4c_s(1 - c_s) - (1 - N_{pl})^2}{4N_{pl}} \end{aligned} \quad (2)$$

Inequality (2) defines the set L^s as

$$L^s = \{(p_l, N_{pl}) | \pi(p_l, N_{pl}, p_{mh}, N_{pl}, c_l, 1) \geq c_s(1 - c_s)\}.$$

We then conclude that sequential rationality is satisfied for the low-cost type when

$$(p_l, N_{pl}) \in L^s.$$

This is illustrated in figure 2⁴.

4.2.2. Sequential Rationality for the High-Cost Type

Sequential rationality for the high-cost type is satisfied when

$$\pi(p_l, N_{pl}, p_{mh}, N_{rh}, c_h, 1) \leq 0.$$

The left-hand side of the inequality is type *h*'s profits from mimicking the prepayment-contract strategy of the low-cost type. As has been mentioned in note 3, we allow the high-cost incumbent to choose his own profit-maximizing period-3 monopoly price. The right-hand side describes the maximal profits that the high-cost firm gets assuming separation. These are zero (since consumers would buy from the entrant), the same as the full-information level of profits.

Let us rewrite the above inequality as follows.

$$(p_l - c_h)N_{pl} + (p_{mh} - c_h)(1 - N_{pl} - p_{mh}) \leq 0. \quad (3)$$

The first term of the left-hand side describes prepayment profits (when the high-cost type duplicates the prepayment-contract strategy of the low-cost type) while the second term describes maximal profits from sales to the residual demand. As with the low-cost type, we allow the high-cost type to optimally choose his period-3 monopoly price. Maximal monopoly profits are as follows.³

$$(p_{mh} - c_h)(1 - N_{pl} - p_{mh}) = \max_p (p - c_h)(1 - N_{pl} - p)$$

where

$$p_{mh} = \frac{1 - N_{pl} + c_h}{2}.$$

Substituting back into (3) we get

$$\begin{aligned} (p_l - c_h)N_{pl} + \frac{(1 - N_{pl} - c_h)^2}{4} &\leq 0 \\ \Rightarrow p_l &\leq \frac{4c_h N_{pl} - (1 - N_{pl} - c_h)^2}{4N_{pl}}. \end{aligned} \quad (4)$$

Inequality (4) defines the set H^E as

$$H^E = \{(p_l, N_{pl}) \mid \pi(p_l, N_{pl}, p_{mh}, N_{mh}, c_h, 1) \leq 0\}.$$

We then conclude that sequential rationality is satisfied for the high-cost type when

$$(p_l, N_{pl}) \in H^E.$$

This is illustrated in figure 4.

Combining the previous results we obtain the following lemma which describes the set of prepayment-contract strategies that reveal low cost to consumers.

Lemma 1. *The set S^E of prepayment-contract strategies that support sequential separating equilibria is*

$$\{(p_l, N_{pl}) \mid (p_l, N_{pl}) \in L^E \cap H^E\}.$$

Lemma 1 is illustrated in figure 5.⁶

The following theorem establishes existence of sequential separating equilibria.

Theorem 1. *The set $L^E \cap H^E$ of sequential separating equilibria is non-empty.*

Proof: See appendix A1.

4.3. Undominated Sequential Separating Equilibria

The previous section has shown that there is a multiplicity of separating equilibria due to the arbitrariness of off-the-equilibrium-path beliefs. The lack of restrictions on beliefs following zero-probability events may lead to non-sensible outcomes being supported by strategies that are sequentially rational and beliefs that are Bayes-consistent.

By applying the criterion that we have stated in chapter 4, we can eliminate many separating equilibria. This criterion suggests that consumers must assign zero probability to a type for which (p_i, N_{pi}) is weakly dominated as long as (p_i, N_{pi}) is not dominated for the other type.

From the construction of H^E we know that for all $(p_i, N_{pi}) \in H^E$

$$\pi(p_i, N_{pi}, p_{mh}, N_{rh}, c_h, 1) \leq 0$$

which implies that (p_i, N_{pi}) is weakly dominated for the high-cost type, but not for the low-cost one as long as $(p_i, N_{pi}) \in \text{int}(L^E)$. According to our criterion, therefore, buyer beliefs must assign $\rho(p_i, N_{pi}) = 1$ to any $(p_i, N_{pi}) \in L^E \cap H^E$ where $\pi(p_i, N_{pi}, p_{mh}, N_{rh}, c_h, 1) > c_h(1 - c_h)$, and the low-cost type should maximize profits given these beliefs. These considerations lead to the following lemma.

Lemma 2. In any undominated sequential separating equilibrium of the model with the efficient-serving rule, the low-cost type chooses (p_l^E, N_{pl}^E) such that

$$(p_l^E, N_{pl}^E) \in \underset{(p_l, N_{pl}) \in S_p \times S_H}{\operatorname{argmax}} \pi(p_l, N_{pl}, p_{ml}, N_{rl}, c_l, 1)$$

subject to the constraints that $(p_l, N_{pl}) \in H^E$ and $p_{ml} \geq p_l$.

The next task is to characterize the set of (p_l^E, N_{pl}^E) pairs that obeys lemma 2. We will show that the problem of the low-cost type has a unique solution. This solution is stated in the following theorem.

Theorem 2. The model with the efficient-serving rule contains a unique undominated sequential separating equilibrium collection $\{(p_l^E, N_{pl}^E), p_{ml}^E, B(p_l^E, N_{pl}^E), p(p_l^E, N_{pl}^E)\}$ such that

$$p_l^E - p_{ml}^E = \frac{1}{2} \left(1 + c_h - \sqrt{2c_h^2 - 2c_h + 1} \right)$$

$$N_{pl}^E = -c_h + \sqrt{2c_h^2 - 2c_h + 1}$$

$$p(p_l^E, N_{pl}^E) = 1$$

$$B(p_l^E, N_{pl}^E) = 1.$$

Proof: See appendix A2.

4.4. Description of the Result and Intuition

As is shown in the appendix, the unique sensible solution of the low-cost type's maximization problem is a pair (p_l^E, N_p^E) which lies on the intersection of the two constraints in the incumbent's maximization problem. That is, the solution lies on the boundary of the set H^E and has the property that $p_l^E = p_m^E$, i.e., the prepayment is equal to the period-3 monopoly price. The immediate implication of this property is that there is no rationing which means that all buyers pay the same price.

There are two ways to interpret this result. First, $1 - p_l^E$ buyers pay in period 1 a prepayment price of p_l^E and then get the product in period 3. Second, only $N_p^E < 1 - p_l^E$ customers prepay p_l^E in period 1 while N_r^E buyers, forming the residual demand, purchase the product in period 3 by paying $p_m^E = p_l^E$. The residual demand is equal to

$$N_r^E = 1 - N_p^E - p_m^E = 1 - N_p^E - p_l^E$$

so that the total demand is equal to

$$N_p^E + N_r^E = 1 - p_l^E.$$

In other words, all consumer types with reservation prices $R \in [p_l^E, 1]$ pay the same price of p_l^E . Those with reservation prices in $[Q^E, 1]$, where $Q^E = 1 - N_p^E$, pre-purchase the product by paying p_l^E while the rest purchase the product in period 3 by paying the same price. Furthermore, the prepayment p_l^E is lower than c_h , the high-cost type's cost. Since the high-cost type chooses a period-3 monopoly price which is higher than c_h , inequality (3) implies that this type will be prevented from mimicking the low-cost type's prepayment strategy only if the low-cost type chooses a prepayment sufficiently lower than c_h . Of course, the choice of the prepayment should be accompanied by an appropriate choice of the number of contracts

to be signed. The above remarks are illustrated in figure 7.

Ideally, what the low-cost incumbent would like to do is to set N_{pi}^E equal to $1 - c_h$ and charge a prepayment equal to c_h (point I in figure 7). This would imply, however, that $p_{mi}^E = c_h/2 < p_i^E$ which is not possible under rationing assumptions. Hence, point E is the best that the low-cost incumbent can do given that he has to satisfy the constraints.

Let us examine now why the incumbent would not choose a point like J in figure 8 (figure 8 reproduces points E and I of figure 7) which is associated with rationing. Increasing p_i^* by moving along the boundary of H^E leads to an increase in N_{pi}^* and thus to an increase in profits from prepayments equal to the area of $p_i^* p_i^E EFGJ p_i^*$. There are two more effects to be considered, however.

First, part of the increase in these profits, equal to the area of EFGH, is outweighed by a decrease in profits from sales to the residual demand. This decrease is due to the decline in the residual demand (N_{ri}^*) caused by the initial increase in N_{pi}^* , keeping p_{mi}^* constant.

Second, the increase in N_{pi}^* leads to a decrease in the period-3 monopoly price which in turn increases the residual demand. As we can see from the figure, the net effect on profits of the decrease in period-3 monopoly price and the resulting increase in residual demand (which is found by adding up the two shaded areas) is of second order compared to the net increase in prepayment profits which is equal to the area of $p_i^* p_i^E EFGJ p_i^*$. Thus, it is optimal for the low-cost incumbent to move towards E.

It is interesting to note that the fact that the low-cost incumbent signals his type by building up a customer base consisting of the highest-reservation-price individuals does not allow him to exploit his monopolistic position in the last period by charging a monopoly price equal to the full-information monopoly price of $1/2$. Charging such a price in period 3 requires that a sufficient number of consumers with reservation prices greater than $1/2$ not sign contracts

in period 1 and be willing to purchase in period 3. More precisely, setting a monopolistic price of $1/2$ requires that N_{μ}^R be equal to zero. This is not, however, an optimal behaviour if the low-cost incumbent is to reveal his type by rendering unprofitable any attempts of the high-cost type to mimic his pricing policy.

Successful signalling requires the low-cost type to sign a sufficiently high number of contracts by attracting the highest-reservation price consumers among those who are willing to prepay. This behaviour then decreases the likelihood of charging a high monopoly price in the last period since those consumers who would be most likely willing to pay such a price have already prepaid. Indeed, the higher the number of contracts signed, the lower the period-3 monopoly price is. As we have seen, it is optimal for the low-cost incumbent to sign that number of contracts which drives the monopoly price down to the prepayment.

5. The Proportional-Serving Scheme

5.1. Description of the Proportional-Serving Scheme

Under this serving scheme, all consumers have the same probability of prepaying or, in other words, being served first. Let us assume that p_i is the prepayment that the low-cost type would choose in the separating equilibrium. Then, as is shown in figure 9, consumers with reservation prices in $[p_i, 1]$ would be willing to prepay.

If, however, rationing occurs in the separating equilibrium then only N_{μ} will have the chance to prepay, where

$$N_{\mu} < 1 - p_i$$

Then, the probability of not being able to prepay when the price is p_i is

$$\frac{(1 - p_i) - N_{it}}{1 - p_i}$$

Consumers who do not prepay will be able to buy the product only if they accept to pay a higher period-3 monopoly price which is equal, say, to p_{m3} . Total demand at this price is $1 - p_{m3}$. Hence, the residual demand is equal to

$$N_{it} = (1 - p_{m3}) \frac{1 - p_i - N_{it}}{1 - p_i}$$

This serving rule is not efficient for buyers since some buyers with reservation prices below p_{m3} buy the good because they are fortunate enough to obtain the bargain price p_i .

5.2. Sequential Separating Equilibria

Since our definition of a separating equilibrium is the same as in 4.2, there is no need to repeat it here. So we start directly with investigating the optimality conditions for the two incumbent types.

5.2.1. Sequential Rationality for the Low-Cost Incumbent

Condition 1 in our definition of a separating equilibrium is satisfied for the low-cost type when

$$\begin{aligned}
 p_l N_{pl} + p_{ml} N_{rl} &\geq c_o(1 - c_o) \\
 \Rightarrow p_l N_{pl} + p_{ml}(1 - p_{ml}) \frac{1 - p_l - N_{pl}}{1 - p_l} &\geq c_o(1 - c_o). \quad (5)
 \end{aligned}$$

The left-hand side describes profits from prepayments plus the maximal profits from sales to the residual demand. That is

$$p_{ml}(1 - p_{ml}) \frac{1 - p_l - N_{pl}}{1 - p_l} = \max_p p(1 - p) \frac{1 - p_l - N_{pl}}{1 - p_l}$$

where $p_{ml} = 1/2$. As opposed to the case of the efficient-serving rule, the period-3 monopoly price is constant and independent of N_{pl} . This implies that the low-cost type is committed to charge a period-3 monopoly price equal to $1/2$ (that is, the full-information monopoly price) regardless of his choice of N_{pl} .

By substituting for p_{ml} we get

$$\begin{aligned}
 p_l N_{pl} + \frac{1 - p_l - N_{pl}}{4(1 - p_l)} &\geq c_o(1 - c_o) \\
 \Rightarrow 4(1 - N_{pl})p_l N_{pl} + (1 - p_l - N_{pl}) &\geq 4c_o(1 - c_o)(1 - p_l) \\
 \Rightarrow [4(1 - p_l)p_l - 1]N_{pl} + [1 - 4c_o(1 - c_o)](1 - p_l) &\geq 0 \\
 \Rightarrow N_{pl} &\leq (1 - 2c_o)^2 \frac{1 - p_l}{(1 - 2p_l)^2}. \quad (6)
 \end{aligned}$$

Inequality (6) defines the set L^P as

$$L^P = \{(p_l, N_M) | \pi(p_l, N_M, p_M, N_M, c_l, 1) \geq c_a(1 - c_a)\}.$$

We then conclude that sequential rationality is satisfied for the low-cost type when

$$(p_l, N_M) \in L^P.$$

This is illustrated in figure 10.

5.2.2 Sequential Rationality for the High-Cost Incumbent

The high-cost type will be prevented from mimicking the prepayment-contract strategy of the low-cost type if his profits from prepayments plus the maximal profits from sales to the residual demand are less than or equal to zero (i.e., the high-cost type's full-information profit level):

$$(p_l - c_h)N_M + (p_{mh} - c_h)(1 - p_{mh}) \frac{1 - p_l - N_M}{1 - p_l} \leq 0 \quad (7)$$

where

$$(p_{mh} - c_h)(1 - p_{mh}) \frac{1 - p_l - N_M}{1 - p_l} = \max_p (p - c_h)(1 - p) \frac{1 - p_l - N_M}{1 - p_l}.$$

Maximization yields

$$p_{mh} = \frac{1 + c_h}{2}.$$

Substituting back into (7) we get

$$\begin{aligned} (p_l - c_h)N_{pl} + (1 - c_h)^2 \frac{1 - p_l - N_{pl}}{4(1 - p_l)} &\leq 0 \\ \Rightarrow N_{pl} &\geq \frac{(1 - c_h)^2 (1 - p_l)}{(1 - c_h)^2 - 4(1 - p_l)(p_l - c_h)}. \end{aligned} \quad (8)$$

Using (8) we can define the set H^p as follows.

$$H^p = \{(p_l, N_{pl}) | \pi(p_l, N_{pl}, p_{mh}, N_{mh}, c_h, 1) \leq 0\}.$$

H^p is illustrated in figure 11. For any $(p_l, N_{pl}) \in H^p$, the high-cost type would never choose such a pair.

Combining the previous results we obtain the following lemma which describes the set of prepayment-contract strategies that reveal low cost to the buyers.

Lemma 3. *The set S^p of prepayment-contract strategies that support sequential separating equilibria is*

$$\{(p_l, N_{pl}) | (p_l, N_{pl}) \in L^p \cap H^p\}.$$

Lemma 3 is illustrated in figure 12.

The following theorem establishes existence of sequential separating equilibria.

Theorem 3. *The set $L^* \cap H^*$ of sequential separating equilibria is non-empty.*

Proof: See appendix A3.

5.3. Undominated Sequential Separating Equilibria

Using the same arguments as in 3.3, all $(p_i, N_{pi}) \in H^*$ are weakly dominated for the high-cost type, but not for the low-cost one as long as $(p_i, N_{pi}) \in \text{int}(L^*)$. Hence, buyer beliefs must assign $\rho(p_i, N_{pi}) = 1$ to any $(p_i, N_{pi}) \in L^* \cap H^*$ where $\pi(p_i, N_{pi}, p_m, N_m, c_h, 1) > c_h(1 - c_s)$, and the low-cost type should maximize profits given these beliefs. Lemma 4 summarizes this result while theorem 4 characterizes the unique undominated sequential separating equilibrium.

Lemma 4. *In any undominated sequential separating equilibrium of the model with the proportional-serving rule, the low-cost type chooses (p_l^*, N_{pl}^*) such that*

$$(p_l^*, N_{pl}^*) \in \underset{(p_l, N_{pl}) \in p_p \times s_H}{\text{argmax}} \pi(p_l, N_{pl}, p_m, N_m, c_l, 1)$$

and $(p_l^*, N_{pl}^*) \in H^*$.

Theorem 4. *The model with the proportional-rationing rule accepts a unique undominated sequential separating equilibrium collection $\{(p_l^*, N_{pl}^*), p_m^*, B(p_l^*, N_{pl}^*), p(p_l^*, N_{pl}^*)\}$ where*

$$p_1^P = \frac{1 - c_1}{2}$$

$$N_{\mu}^P = \frac{c_A^3 - c_A^2 - c_A + 1}{14c_A^3 + 4c_A - 2}$$

$$p_{\mu}^P = \frac{1}{2}$$

$$p(p_1^P, N_{\mu}^P) = 1$$

$$B(p_1^P, N_{\mu}^P) = 1.$$

Proof: See appendix A4.

5.4. Description of the Result and Intuition

Figure 13 illustrates the result stated in theorem 4. Contrary to what we showed for the case of the efficient-serving scheme, the proportional-serving scheme gives rise to rationing in the separating equilibrium. N_{μ}^P consumers pre-purchase the product by paying a prepayment price of p_1^P while N_{μ}^P consumers wait until period 3 in order to get one unit of the product at the monopoly price of p_{μ}^P . Rationing is also reflected in the fact that N_{μ}^P is less than $1 - p_1^P$, the total demand for contracts at the price of p_1^P .

It is interesting to note that the period-3 monopoly price is equal to the monopoly price that the low-cost type would charge if there were full information, i.e., $p_{\mu}^P = 1/2$. This is due to the fact that the residual demand is just the total demand pivoted (around its vertical intercept)

to the left by a factor reflecting the probability of not being able to prepay. In other words, every consumer who is part of the total demand has the same chance of being a part of the residual demand as well. This characteristic prompts the low-cost type to adopt that pricing policy towards the residual demand that he would also adopt towards the total demand under conditions of full information.

Alternatively stated, in the case of the proportional-serving pattern, every consumer, regardless of his reservation price, has the same probability of signing a contract. Since, therefore, the residual demand will be a mix of low and high-reservation-price consumers, the low-cost incumbent has the flexibility of charging the full-information monopoly price in period 3.

6. Equilibrium Effects of Changes in c_h

6.1. Effects on the Size of Equilibrium Prepayments, p_1^E and p_1^F

The higher the high-cost type's cost, the larger the set of (p_1, N_H) pairs that support sequential separating equilibria and the more attractive (from the point of view of the low-cost incumbent) the choice of the unique equilibrium (p_1^E, N_H^E) pair will be. Given that the prepayment and the period-3 monopoly price are equal when the serving rule is the efficient one and, furthermore, every consumer who purchases the product pays the same prepayment price, relaxing the signalling constraint (because of a higher c_h) is equivalent to saying that the low-cost firm can signal its type by setting a higher prepayment price. A higher prepayment in turn implies a lower total demand.

In the case of the proportional-serving scheme, however, there is rationing in equilibrium

which implies that some customers pay a prepayment that is smaller than the period-3 monopoly price (which is equal to the full-information monopoly price). Then, relaxing the signalling constraint (because of a higher c_h) implies that the low-cost type's pricing policy should be oriented towards a higher residual demand and, therefore, more consumers buying at the period-3 full-information monopoly price. The residual demand will increase if the number of customers who prepay ($N_{p_i}^P$) decreases. A lower $N_{p_i}^P$ in turn implies that the prepayment p_i^P should decrease along the boundary of H^P . As a result, a higher c_h allows the low-cost firm to reduce the number of contracts that he offers and increase sales to the more profitable residual demand.

Figure 14, which depicts the change in $p_i^P - p_i^E$ as c_h changes, illustrates the previous points. When c_h is relatively low and the serving scheme is the proportional one, the low-cost type signals its cost by forming a relatively large customer base ($N_{p_i}^P$). A large $N_{p_i}^P$ implies a large p_i^P as well. The key to this result is the fact that the period-3 monopoly price is fixed at $1/2$. Then the only thing that the low-cost type can do to signal his cost is to increase his supply of contracts and limit the residual demand. If the serving rule is the efficient one, however, then there is no rationing and the number of customers paying the prepayment price is determined by the (total) demand curve. Then, signalling through a large offer of contracts implies a low prepayment along the demand curve.

As c_h increases, the lesser need for signalling is reflected in a higher prepayment when the serving scheme is the efficient one and in a lower $N_{p_i}^P$ and p_i^P when the serving scheme is the proportional one.

6.2. Effects on Equilibrium Profits Under Both Serving Schemes

Figure 15 depicts the variation (as c_A changes) in the difference between the equilibrium profits under the proportional-serving scheme (π^P) and those under the efficient-serving scheme (π^E).^{7,8} Figure 16 decomposes this variation in two parts. The top diagram illustrates the change in the difference between the equilibrium prepayment profits ($\pi_p^P - \pi_p^E$) while the bottom part illustrates the change in the difference between the equilibrium residual-demand profits ($\pi_r^P - \pi_r^E$).

From the top diagram in figure 16, we can see that when c_A is relatively high, the period-1 equilibrium profits from prepayments under the proportional-serving scheme are lower than the analogous profits for the case of the efficient-serving scheme. On the contrary, the bottom part of figure 16 shows that the equilibrium profits from sales to the residual demand under the proportional-serving scheme are larger than the corresponding profits for the case of the efficient-serving scheme. The explanation lies in the fact that (as has been explained in section 6.1) when c_A is relatively large, the low-cost incumbent signals his type through a smaller prepayment (and thus a smaller number of period-1 contracts and a larger residual demand) for the case of the proportional-serving scheme while he signals his type through a higher prepayment (along the total demand curve) for the case of the efficient-serving scheme. If c_A takes a value in (0.537, 1) then the advantage of the proportional-serving scheme in terms of profits from sales to the residual demand is more than offset by the advantage of the efficient-serving scheme in terms of period-1 prepayment profits. Then, total profits under the efficient-serving scheme are higher than total profits under the proportional-serving scheme (figure 15).

If c_A takes values that are very close to 0.33, the proportional-serving scheme yields prepayment profits that are higher than those under the efficient-serving scheme while the reverse is true for profits from sales to the residual demand. This is due to the fact that when

c_h is relatively low, the low-cost incumbent signals his type through a large number of period-1 contracts which imply a large prepayment under the proportional-serving scheme but a small one under the efficient-serving scheme. Figure 15 reveals that (for a low c_h) the advantage of the proportional-serving scheme in terms of prepayment profits outweighs the advantage of the efficient-serving scheme in terms of profits from sales to the residual demand. Thus, overall, the proportional-serving scheme yields larger profits than the efficient-serving scheme in the case where c_h is relatively low.

This advantage of the proportional-serving scheme fades away when c_h increases towards 0.537 so that the positive difference between profits under the proportional-serving scheme and profits under the efficient-serving scheme starts declining.

Although figure 15 may suggest that the incumbent's expected profits can be used as a criterion for the choice of the serving pattern, the actual shape of the residual demand cannot be specified generally from a priori reasoning. It is an important and complicated marketing problem which needs specific empirical investigation. If, for example, the total demand represents the sum of the inelastic demands of heterogeneous consumers all of whom wish to purchase one unit of the good (provided the price is below their reservation value) then firms cannot easily influence the manner in which demand is rationed. The residual demand curve simply depends on the arrival process of consumers in period 1. This process may be influenced by the location of consumers with respect to the incumbent firm, the speed with which consumers obtain information about the incumbent's period-1 pricing policy, transportation costs and other similar factors that are mostly unaffected by firm behaviour. In such a case, the proportional-serving rule seems most appropriate to choose seems it amounts to an assumption of symmetric treatment of consumers. On the other hand, the efficient-serving rule assumes that consumers with the highest reservation prices are always

served first which is not necessarily true in the absence of a resale market. Thus, although a sufficiently high c_h may imply that the efficient-serving rule yields higher expected profits than the proportional-serving rule, the above considerations may force the incumbent to use the proportional-serving rule.

7. Conclusion

We have analyzed an extension of chapter 4 by introducing the assumption of heterogeneous buyers indexed by their reservation prices. Buyer heterogeneity gives rise to a unique undominated separating equilibrium in which the low-cost type is prepared to sign a number of contracts requiring customers to pre-purchase one unit of the product and pay a prepayment price. Furthermore, the number of contracts offered by the low-cost type may be less than the number of buyers who are willing to sign and prepay. Thus, rationing may emerge in equilibrium and this depends on the types of consumers that prepay.

Appendix

A1. *Proof of Theorem 1.* From the definition of L^E we conclude that $\exists (p_i, N_{pi}) \in L^E$ such that

$\pi(p_i, N_{pi}, p_{mb}, N_{mb}, c_h, 1) = c_h(1 - c_h)$ which we can write as

$$p_i N_{pi} + \frac{(1 - N_{pi})^2}{4} = c_h(1 - c_h).$$

To prove that the set $L^E \cap H^E$ is non-empty, it is sufficient to prove that

$$\pi(p_i, N_{pi}, p_{mb}, N_{mb}, c_h, 1) < 0.$$

Assume on the contrary that

$$\pi(p_i, N_{pi}, p_{mb}, N_{mb}, c_h, 1) \geq 0$$

or

$$(p_i - c_h)N_{pi} + \frac{(1 - N_{pi} - c_h)^2}{4} \geq 0. \quad (A1)$$

By the definition of H^E and by construction we know that

$$\frac{c_h(1 - c_h)}{4} - (p_i - c_h)N_{pi} + \frac{(1 - N_{pi} - c_h)^2}{4} \leq 0$$

$$p_i N_{pi} + \frac{(1 - N_{pi}')^2}{4} - c_s(1 - c_s) = 0. \quad (A3)$$

Combining (A1) and (A2) we have that

$$(p_i' - c_s)N_{pi}' + \frac{(1 - N_{pi}' - c_s)^2}{4} - (p_i - c_s)N_{pi} - \frac{(1 - N_{pi} - c_s)^2}{4} \geq 0.$$

Subtracting (A3) we get

$$\begin{aligned} (p_i' - c_s)N_{pi}' + \frac{(1 - N_{pi}' - c_s)^2}{4} - (p_i - c_s)N_{pi} - \frac{(1 - N_{pi} - c_s)^2}{4} - p_i N_{pi}' - \frac{(1 - N_{pi}')^2}{4} \\ + c_s(1 - c_s) \geq 0. \end{aligned}$$

Going through the algebra yields the following.

$$p_i N_{pi} + \frac{(1 - N_{pi}')^2}{4} \leq c_s(1 - c_s) - \frac{1}{2} c_s(N_{pi}' - N_{pi}). \quad (A4)$$

As we can see from figure 2, for any (p_i, N_{pi}) such that $\pi(p_i, N_{pi}, p_{mi}, N_{mi}, c_p, 1) = c_s(1 - c_s)$, we can find $(p_i, N_{pi}) \in \text{int}(L^s)$ with $N_{pi} < N_{pi}'$. For any such N_{pi} , (A4) leads to a contradiction since it is true that, for any $(p_i, N_{pi}) \in L^s$,

$$p_i N_{pi} + \frac{(1 - N_{pi}')^2}{4} \geq c_s(1 - c_s).$$

Hence, $\pi(p_l, N_{pl}, p_{ml}, N_{dl}, c_h, 1) < 0$ and $L^E \cap H^E \neq \emptyset$.

A2. *Proof of Theorem 2.* The low-cost type's decision problem can be stated as follows.

$$\begin{aligned} \max_{(p_l, N_{pl}) \in S_p \times S_N} & \pi(p_l, N_{pl}, p_{ml}, N_{dl}, c_l, 1) \\ \text{s.t.:} & (p_l, N_{pl}) \in H^E \\ & p_{ml} \geq p_l \end{aligned}$$

Substituting for the profit function and the constraints we get

$$\begin{aligned} \max_{(p_l, N_{pl}) \in S_p \times S_N} & \left[p_l N_{pl} + \frac{(1 - N_{pl})^2}{4} \right] \\ \text{s.t.:} & \frac{4c_h N_{pl} - (1 - N_{pl} - c_h)^2}{4N_{pl}} - p_l \geq 0 \\ & \frac{1 - N_{pl}}{2} - p_l \geq 0. \end{aligned}$$

The Lagrangian is defined as follows.

$$\Lambda^E = p_l N_{pl} + \frac{(1 - N_{pl})^2}{4} + \lambda_1 \left[\frac{4c_h N_{pl} - (1 - N_{pl} - c_h)^2}{4N_{pl}} - p_l \right] + \lambda_2 \left(\frac{1 - N_{pl}}{2} - p_l \right).$$

The first-order conditions for a maximum are

$$\begin{aligned}
\Lambda_{p_l}^E - N_{pl} - \lambda_1^E - \lambda_2^E &= 0 \\
\Lambda_{N_{pl}}^E - p_l - \frac{1 - N_{pl}}{2} + \lambda_1^E - \frac{2(1 - N_{pl} - c_h)N_{pl} + (1 - N_{pl} - c_h)^2}{4N_{pl}^2} - \frac{\lambda_2^E}{2} &= 0 \\
\Lambda_{\lambda_1^E}^E - \frac{4c_h N_{pl} - (1 - N_{pl} - c_h)^2}{4N_{pl}} - p_l &\geq 0 \quad \wedge \quad \lambda_1^E \Lambda_{\lambda_1^E}^E = 0 \\
\Lambda_{\lambda_2^E}^E - \frac{1 - N_{pl}}{2} - p_l &\geq 0 \quad \wedge \quad \lambda_2^E \Lambda_{\lambda_2^E}^E = 0.
\end{aligned}$$

Let us now consider the following cases.

Case 1: $\lambda_1^E > 0$ and $\lambda_2^E = 0$. This is the case of rationing since $\lambda_2^E = 0$ implies that

$$\Lambda_{\lambda_2^E}^E = \frac{1 - N_{pl}}{2} - p_l > 0 \Rightarrow p_{ml} > p_l$$

Since $\lambda_2^E = 0$ and $\lambda_1^E > 0$, we have that $N_{pl} = \lambda_1^E > 0$. Then from the first-order condition with respect to N_{pl} we have that

$$p_l = \frac{1 - N_{pl}}{2} - \frac{2(1 - N_{pl} - c_h)N_{pl} + (1 - N_{pl} - c_h)^2}{4N_{pl}^2}. \quad (A5)$$

Furthermore, since $\lambda_1^E > 0$, the third first-order condition yields

$$p_l = \frac{4c_h N_{pl} - (1 - N_{pl} - c_h)^2}{4N_{pl}}. \quad (A6)$$

Equating (A5) and (A6) we get

$$\begin{aligned} 2(1 - N_{pl})N_{pl} - 2(1 - N_{pl} - c_h)N_{pl} - (1 - N_{pl} - c_h)^2 - 4c_h N_{pl} - (1 - N_{pl} - c_h)^2 \\ \Rightarrow 2(1 - N_{pl})N_{pl} - 2(1 - N_{pl})N_{pl} + 2c_h N_{pl} - 4c_h N_{pl} \\ \Rightarrow 2c_h N_{pl} = 0. \end{aligned}$$

This is not acceptable since $c_h > 0$ and $N_{pl} = \lambda_1^E > 0$.

Case 2: $\lambda_1^E = 0$ and $\lambda_2^E > 0$. Since $\lambda_1^E = 0$, we have that $N_{pl} = \lambda_2^E > 0$. Then, the first-order condition with respect to N_{pl} yields

$$p_l = \frac{1 - N_{pl}}{2} + \frac{N_{pl}}{2} = \frac{1}{2}.$$

On the other hand, using $\lambda_2^E > 0$, the first-order condition with respect to λ_2^E yields

$$\frac{1 - N_{pl}}{2} - p_l = \frac{1}{2} - \frac{N_{pl}}{2} = 0.$$

This is not possible since $N_{pl} = \lambda_2^E > 0$.

Case 3: $\lambda_1^E = \lambda_2^E = 0$. This implies that $N_{pl} = 0$ which contradicts our initial assumption.

Case 4: $\lambda_1^E > 0$ and $\lambda_2^E > 0$. Both constraints are binding which yields

$$p_l = \frac{4c_h N_{pl} - (1 - N_{pl} - c_h)^2}{4N_{pl}}$$

$$p_l = \frac{1 - N_{pl}}{2}.$$

By equating the right-hand sides we get

$$2N_{pl}(1 - N_{pl}) - 4c_h N_{pl} - (1 - N_{pl} - c_h)^2$$

$$\Rightarrow 2N_{pl} - 2N_{pl}^2 - 4c_h N_{pl} - 1 - N_{pl}^2 - c_h^2 + 2N_{pl} + 2c_h - 2c_h N_{pl}$$

$$= N_{pl}^2 + 2c_h N_{pl} - (1 - c_h)^2 = 0.$$

The roots of this polynomial are given by the following.

$$N_{pl}^{1,2} = \frac{-2c_h \pm \sqrt{4c_h^2 - 4(1 - c_h)^2}}{2}$$

$$= \frac{-2c_h \pm \sqrt{8c_h^2 - 8c_h + 4}}{2}$$

$$= -c_h \pm \sqrt{2c_h^2 - 2c_h + 1}.$$

The acceptable root is

$$N_{pl}^x = -c_h + \sqrt{2c_h^2 - 2c_h + 1}.$$

This implies that

$$p_l^E - p_{ml}^E = \frac{1 - N_{pl}^E}{2} = \frac{1}{2} \left(1 + c_h - \sqrt{2c_h^2 - 2c_h + 1} \right) < c_h.$$

The residual demand is

$$\begin{aligned} N_H^E - Q^E - p_{ml}^E \\ &= 1 - N_{pl}^E - p_{ml}^E \\ &= \frac{1}{2} \left(1 + c_h - \sqrt{2c_h^2 - 2c_h + 1} \right) \end{aligned}$$

and total demand is equal to

$$N_{pl}^E + N_H^E = \frac{1}{2} \left(1 - c_h + \sqrt{2c_h^2 - 2c_h + 1} \right).$$

A3. Proof of Theorem 3. The procedure to prove this theorem is the same as the one used for the proof of theorem 1. From the definition of L^P , $\exists (p_l', N_{pl}') \in L^P$ such that

$$\pi(p_l', N_{pl}', p_{ml}', N_H', c_h, 1) \geq c_o(1 - c_o)$$

which can be rewritten as

$$p_l' N_{pl}' + \frac{1 - p_l' - N_{pl}'}{4(1 - p_l')} = c_o(1 - c_o).$$

To prove that the set $L^P \cap H^P$ is non-empty, it is sufficient to prove that

$$\pi(p_l', N_{pl}', p_{ml}', N_H', c_h, 1) < 0.$$

Assume on the contrary that

$$\pi(p_i, N_{pi}, p_{ai}, N_{ai}, c_a, 1) \geq 0$$

or

$$(p_i - c_a)N_{pi} + (1 - c_a)^2 \frac{1 - p_i - N_{pi}}{4(1 - p_i)} \geq 0. \quad (A7)$$

By the definition of H^* and by construction we know that

$$(p_i - c_a)N_{pi} + (1 - c_a)^2 \frac{1 - p_i - N_{pi}}{4(1 - p_i)} \leq 0 \quad (A8)$$

$$p_i N_{pi} + \frac{1 - p_i - N_{pi}}{4(1 - p_i)} - c_a(1 - c_a) = 0. \quad (A9)$$

Combining (A7) and (A8) we have that

$$(p_i - c_a)N_{pi} + (1 - c_a)^2 \frac{1 - p_i - N_{pi}}{4(1 - p_i)} - (p_i - c_a)N_{pi} - (1 - c_a)^2 \frac{1 - p_i - N_{pi}}{4(1 - p_i)} \geq 0.$$

Subtracting (A9) we get

$$(p_i - c_a)N_{pi} + (1 - c_a)^2 \frac{1 - p_i - N_{pi}}{4(1 - p_i)} - (p_i - c_a)N_{pi} - (1 - c_a)^2 \frac{1 - p_i - N_{pi}}{4(1 - p_i)}$$

$$-p_i N_{pi} - \frac{1-p_i-N_{pi}}{4(1-p_i)} + c_s(1-c_s) \geq 0.$$

Going through the algebra yields the following.

$$p_i N_{pi} + \frac{1-p_i-N_{pi}}{4(1-p_i)} \leq c_s(1-c_s) - c_h(N_{pi} - N_{pi}) \quad (A10)$$

$$- (c_h^2 - 2c_h) \left[\frac{1-p_i-N_{pi}}{4(1-p_i)} - \frac{1-p_i-N_{pi}}{4(1-p_i)} \right]$$

Figure 10 reveals that for any (p_i, N_{pi}) such that $\pi(p_i, N_{pi}, \varphi_{int}, N_{pi}, c_h, 1) = c_s(1-c_s)$, we can find $(p_i, N_{pi}) \in \text{int}(L^s)$ with $N_{pi} > N_{pi}$ and $p_i < p_i$ such that the last term in (A10) is negative (for instance, $p_i = c_h$ and $N_{pi} = 1 - c_h$). For any such (p_i, N_{pi}) , (A10) leads to a contradiction since it is true that, for any $(p_i, N_{pi}) \in L^s$,

$$p_i N_{pi} + \frac{1-p_i-N_{pi}}{4(1-p_i)} \geq c_s(1-c_s).$$

Hence, $\pi(p_i, N_{pi}, \varphi_{int}, N_{pi}, c_h, 1) < 0$ and $L^s \cap H^s \neq \emptyset$.

A4. *Proof of Theorem 4.* The low-cost type's maximization problem can be stated as follows.

$$\max_{(p_i, N_{pi}) \in \mathcal{S}_p \times \mathcal{S}_N} \left[p_i N_{pi} + \frac{1-p_i-N_{pi}}{4(1-p_i)} \right]$$

$$\text{s.t.: } N_{pi} - \frac{(1-c_h)^2(1-p_i)}{(1-c_h)^2 - 4(1-p_i)(p_i-c_h)} \geq 0.$$

The constraint implies that $(p_i, N_{pi}) \in H^p$. Given that, we can define the Lagrangian as follows.

$$\Lambda^p = p_i N_{pi} + \frac{1 - p_i - N_{pi}}{4(1 - p_i)} + \lambda^p \left[N_{pi} - \frac{(1 - c_h)^2(1 - p_i)}{(1 - c_h)^2 - 4(1 - p_i)(p_i - c_h)} \right]$$

Given the amount of prepayment p_i , the low-cost type will choose the minimal N_{pi} consistent with revealing his type and such that $(p_i, N_{pi}) \in H^p$ in order to expand as much as possible sales to the residual demand. This implies that the optimal (p_i, N_{pi}) pair will be on the boundary of H^p so that the constraint in the maximization problem will be binding. Then, substituting the constraint for N_{pi} in the profit function, we get

$$\begin{aligned} \pi^p &= \frac{p_i(1 - c_h)^2(1 - p_i)}{(1 - c_h)^2 - 4(1 - p_i)(p_i - c_h)} + \frac{1}{4} - \frac{(1 - c_h)^2(1 - p_i)}{4(1 - p_i)[(1 - c_h)^2 - 4(1 - p_i)(p_i - c_h)]} \\ &= \frac{1}{4} + \frac{(1 - p_i)(1 - c_h)^2[4p_i(1 - p_i) - 1]}{4(1 - p_i)[(1 - c_h)^2 - 4(1 - p_i)(p_i - c_h)]} \\ &= \frac{1}{4} - \frac{(1 - 2p_i)^2(1 - c_h)^2}{4(1 - c_h)^2 - 16(1 - p_i)(p_i - c_h)} \end{aligned}$$

Computing the first-order condition with respect to p_i and setting it equal to zero yields the following.

$$4p_i^2 - 4p_i - c_h^2 + 1 = 0. \quad (A11)$$

For a maximum we require that

$$8p_i - 4 < 0$$

$$p_l < \frac{1}{2}.$$

The solutions to (A11) are as follows.

$$p_l^{1,2} = \frac{4 \pm \sqrt{16 - 16(1 - c_h^2)}}{8} \\ = \frac{1 \pm c_h}{2}.$$

The acceptable solution from the second-order condition is

$$p_l^P = \frac{1 - c_h}{2} < \frac{1}{2}$$

implying a total demand for contracts equal to

$$1 - p_l^P = \frac{1 + c_h}{2}.$$

Using our solution for p_l^P we can substitute for the prepayment in the constraint of the maximization problem and get a solution for N_m^P which is given by the following.

$$N_m^P = \frac{c_h^2 - c_h^2 - c_h + 1}{14c_h^2 + 4c_h - 2}.$$

The residual demand is equal to

$$\begin{aligned}
 N_{rl}^P &= (1 - p_{ml}) \frac{1 - p_l^P - N_{rl}^P}{1 - p_l^P} \\
 &= \frac{1 - p_l^P - N_{rl}^P}{2(1 - p_l^P)} \\
 &= \frac{3c_h^3 + 5c_h^2 + c_h - 1}{7c_h^3 + 9c_h^2 + c_h - 1}
 \end{aligned}$$

A5. *The Low-Cost Type's Profits in the Separating Equilibrium.* Profits under the efficient-serving rule are equal to

$$\begin{aligned}
 \pi_l^E &= p_l^E N_{rl}^E + \frac{(1 - N_{rl}^E)^2}{4} \\
 &= \left(\frac{1}{2} + \frac{c_h}{2} - \frac{1}{2} \sqrt{2c_h^3 - 2c_h + 1} \right) \left(-c_h + \sqrt{2c_h^3 - 2c_h + 1} \right) + \frac{1}{4} \left(1 + c_h - \sqrt{2c_h^3 - 2c_h + 1} \right)^2.
 \end{aligned}$$

The algebra yields

$$\pi_l^E = \frac{1}{4} (2c_h \sqrt{2c_h^3 - 2c_h + 1} - 3c_h^2 + 2c_h)$$

which is positive for all c_h in $(0, 1]$.

Similarly, profits under the proportional-serving rule are equal to the following.

$$\begin{aligned}
 \pi_l^P &= p_l^P N_{rl}^P + p_{ml}^P (1 - p_{ml}^P) \frac{1 - p_l^P - N_{rl}^P}{1 - p_l^P} \\
 &= \frac{-7c_h^7 + 5c_h^6 + 59c_h^5 + 71c_h^4 + 15c_h^3 - 13c_h^2 - 3c_h + 1}{196c_h^5 + 308c_h^4 + 72c_h^3 - 56c_h^2 - 12c_h + 4}
 \end{aligned}$$

The denominator of π_i^P becomes zero when $c_a = 0.261$ while $\pi_i^P = 0$ when $c_a = 0.268$. Furthermore, $\pi_i^P > 0$ for any other value of c_a in $(0,1]$.

Notes

1. These serving schemes have been denoted in the economics literature *efficient-rationing* and *proportional-rationing* rules. In the case of the former, those consumers with the highest reservation prices buy from the lower-priced seller whereas under the latter, all consumers have the same chance of buying from the lower-priced seller [for a detailed description of these rules and a general theory of household and market contingent demand see Dixon (1987); Tirole (1988) provides a brief discussion of these rationing rules as well]. Our use of these rules is made in a different context with two specific distinctions. First, in the separating equilibrium there will be a *unique* seller charging two prices, a prepayment price and a higher monopoly price. Second, although the incumbent can impose the *serving* scheme from the outset, *rationing* is endogenous in this model.

2. The assumption that R is distributed uniformly in $[0,1]$ allows us to interpret figure 1 as one depicting the "demand function" for period-1 contracts with each contract giving the right to the customer to pre-purchase one unit of the product at the prepayment price.

3. We should point out, however, that p_{mi} and N_{mi} , $i = l, h$, should be included in the profit functions in order to show the dependence of the overall profits on profits from sales to the residual demand. Nevertheless, we still assume, as will be explained below, that buyer inferences are solely based on the prepayment-contract strategy. As a result, the high-cost type would mimic only the prepayment-contract strategy of the low-cost type while he would choose p_{mh} in order to maximize profits from sales to the residual demand. In the separating equilibrium, however, only type l 's profits will depend on p_{ml} and N_{ml} since only the low-cost

type produces in the separating equilibrium. Furthermore, the sets S_p and S_N should be redefined as follows.

$$S_p = \{p \in \mathbb{R}_+ | p < 1\}$$

and

$$S_N = \{N_p \in \mathbb{R}_+ | N_p < 1\}.$$

4. The function describing the lower boundary of L^2 has two maxima: $N_{pi}^1 = 1 - 2c_e$ occurring at $p_i^1 = c_e$ and $N_{pi}^2 = (1 - 2c_e)$ occurring at $p_i^2 = 1 - c_e$. In the remaining of the chapter we will consider values of c_e in $[0, 0.5]$ so that the first maximum applies. If values of c_e in $(0.5, 1)$ are considered then the diagram in figure 3 applies which, however, does not change the results since what will only constraint the low-cost type when choosing its profit-maximizing, separating (p_i, N_{pi}) pair is the condition describing sequential rationality for the high-cost type. This condition does not depend on c_e .

5. By comparing the monopoly prices set by the two firm types, we can see that $p_{mh} > p_{ml}$. This implies that consumers could infer the incumbent's cost type by observing the monopoly price, disregarding thus the prepayment and the number of period-1 contracts. Nevertheless, we will assume that the determination of the monopoly price occurs after the announcement of (p_i, N_{pi}) has been made and that consumers use only (p_i, N_{pi}) to make their inferences about the incumbent's cost type. By the time the announcement of the monopoly price is made, inferences have been made and consumers have prepaid.

6. The diagrams in this chapter apply for values of c_i that are sufficiently close to c_k . If c_k is much bigger than c_i then the boundaries of L^E and H^E do not intersect. Instead, the boundary of H^E lies above that of L^E . This is illustrated in figure 6.

7. Some steps of the algebra involved in the calculation of these profits are presented in appendix A5.

8. Since we require $(p_i^P, N_{pi}^P) \in H^P$, it must be the case that $p_i^P \leq c_k$. Since, $p_i^P = (1 - c_k)/2$, we require that $c_k \geq 1/3$.

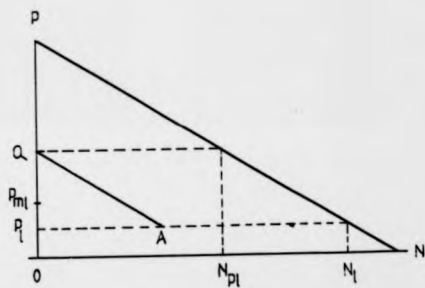


Figure 1. The Efficient-Serving Scheme

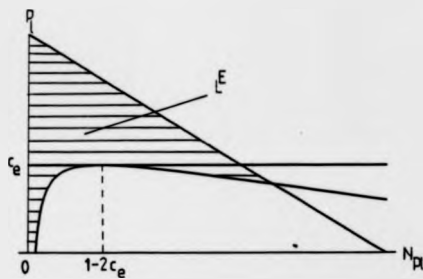


Figure 2. Sequential Rationality for the Low-Cost Type Under the Efficient-Serving Scheme

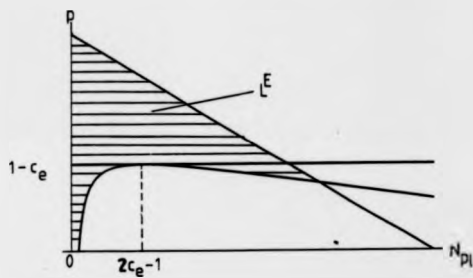


Figure 3. Sequential Rationality for the Low-Cost Type Under the Efficient-Serving Scheme in the Case Where $c_e \in [0.5, 1]$

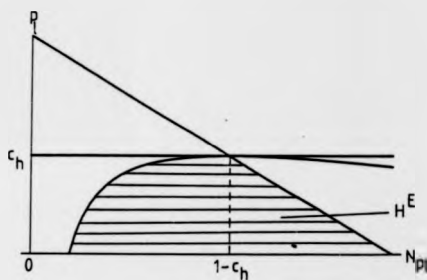


Figure 4. Sequential Rationality for the High-Cost Type Under the Efficient-Serving Scheme

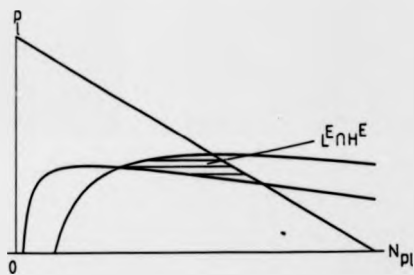


Figure 5. Sequential Separating Equilibria Under the Efficient-Serving Scheme

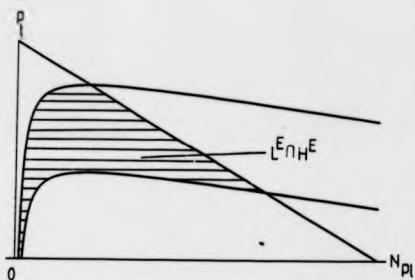
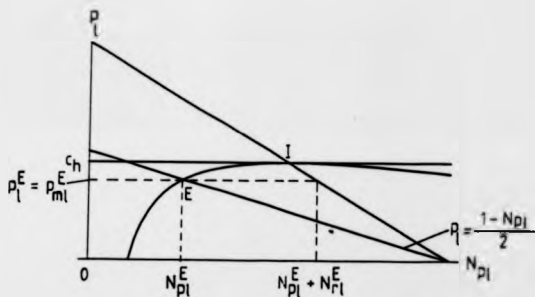


Figure 6. *Sequential Separating Equilibria Under the Efficient-Serving Scheme When c_h is Much Bigger than c_l*



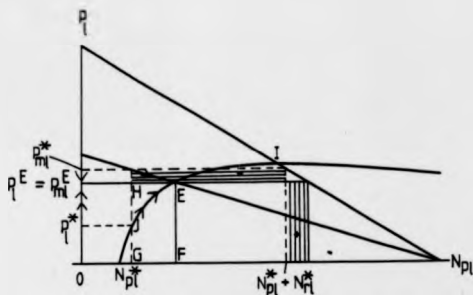


Figure 8. Optimality of the Unique Undominated Sequential Separating Equilibrium
Under the Efficient-Separating Scheme

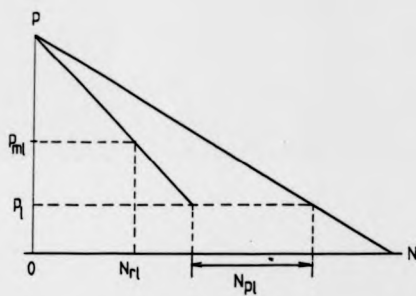


Figure 9. *The Proportional-Serving Scheme*

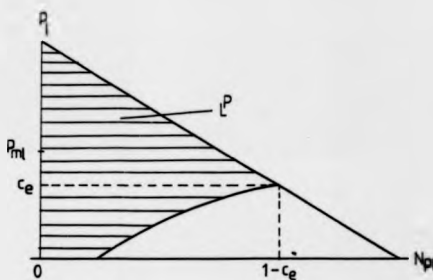


Figure 10. Sequential Rationality for the Low-Cost Type Under the Proportional-Serving Scheme

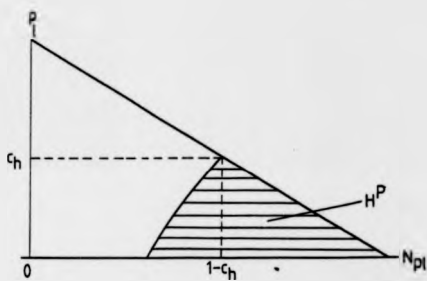


Figure 11. Sequential Rationality for the High-Cost Type Under the Proportional-Serving Scheme

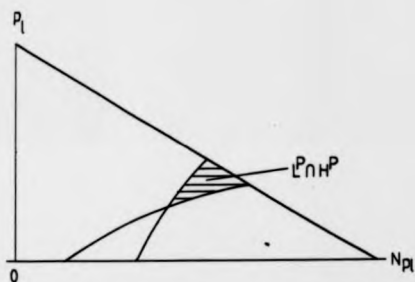


Figure 12. *Sequential Separating Equilibria Under the Proportional-Serving Scheme*



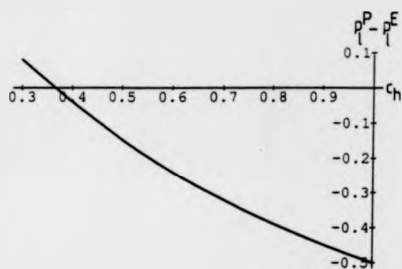


Figure 14. *Equilibrium Effects of Changes in c_h on $p_l^P - p_l^E$*

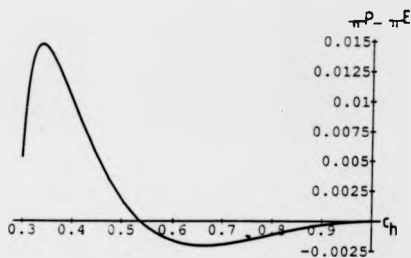


Figure 15. *The Variation in Equilibrium Profits Under the Proportional-Serving Scheme
Minus Equilibrium Profits Under the Efficient-Serving Scheme*

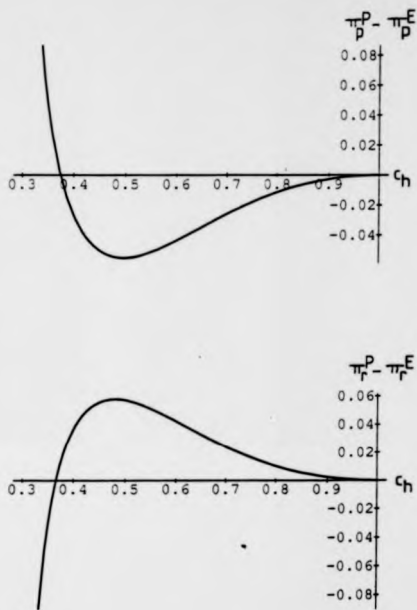


Figure 16. *Decomposition of the Difference Between the Equilibrium Profits Under the Two Serving Schemes in Profits from Prepayments and Profits from Sales to the Residual Demand*

1. Introduction

Pioneering theoretical work on barriers to entry began with Bain (1956) who identified three barriers to entry: (1) absolute cost advantage, (2) economies of large-scale production that requires large capital expenditures, and (3) product differentiation.

An absolute cost advantage can allow an incumbent firm to earn excess profits without fearing entry. The notion of absolute cost advantage as one of the determinants of the conditions of entry has been elaborated under different contexts. Three of these contexts consider strategic behaviour designed to exploit *experience-related cost asymmetries* and *network economies* as well as strategic attempts by incumbents to *raise rivals' costs*.

The implications of experience-related cost advantages for strategic behaviour depend on the type of the competitive environment in which they occur. For instance, Fudenberg et al. (1983) and Harris and Vickers (1985) consider models of innovation in which experience lowers the expected cost of winning a patent. In the Fudenberg et al. model, a firm that is sufficiently ahead in the race for a patent will enjoy a monopoly because rival firms will exclude themselves from the race.

Positive network externalities arise when a good is more valuable to a user the more users adopt the same good or compatible ones. Network economies imply the presence of an economy of scale: either willingness to pay increases or production cost decreases with the

size of the market [Gilbert in Schmalensee and Willig eds. (1989, p. 498)]. Katz and Shapiro (1985, 1986) and Farrell and Saloner (1986) explore the implications of network economies for entry deterrence.

More general is the topic of strategic attempts by incumbent firms to raise rivals' costs. This argument is explored by Salop and Scheffman (1983, 1987) in an environment in which unilateral behaviour by incumbent firms has the consequence of raising industry costs of operations. Salop and Scheffman argue that behaviour intended to increase industry costs can benefit established firms (despite increasing their own costs) because it causes rival firms to reduce their output.¹ An example is ALCOA Corporation's acquisition of bauxite ore deposits which, it was alleged, was designed to raise costs for potential competitors and foreclose entry.

Contracts among incumbents and customers that impose penalties for breach are related to strategies that raise rivals' costs. Aghion and Bolton (1987) provide an example of contracts that facilitate entry prevention. They show that contracts of this type may be signed even though buyers dislike attempts of monopolization of the market.

Aghion and Bolton consider a model in which an incumbent seller, one buyer and the entrant interact for two periods. In period 1 the incumbent and the buyer negotiate a contract with the purpose of preventing an entrant from entering the market. In period 2 there is production and trade. The type of contracts that Aghion and Bolton examine are characterized by two prices. The price of the good when the buyer trades with the incumbent and the price the buyer must pay if he breaks the contract and does not trade with the incumbent. Thus the buyer will trade with the entrant if the entrant charges a price which is lower than the seller's price minus the damages the buyer pays to the seller if he defects from the agreement. These damages, therefore, act as an entry fee the entrant must pay to the seller. Aghion and Bolton

show that the equilibrium contract makes the seller strictly better off and the buyer not worse off from signing it. The buyer is not worse off because, by signing the contract, the incumbent and the buyer form a coalition which behaves like a monopolist with respect to the entrant and thus extracting some of the entrant's rents.

Aghion and Bolton also consider the situation where there are two buyers and one seller. Both buyers are identical and have the same reservation price. Furthermore, the entrant faces a fixed cost of entry. This implies that his average cost is decreasing in the number of customers served and the probability of entry is increasing in the number of customers.

In the more general case of several buyers, Aghion and Bolton argue that

...when one buyer signs a long-term contract with the incumbent, he imposes a negative externality on all other buyers. By locking himself into a long-run relation with the seller, he reduces the size of the entrant's potential market so that, *ceteris paribus*, the probability of entry will be smaller. As a result, the other buyers will have to accept higher prices...the incumbent can exploit this negative externality to extract more (possibly all) surplus out of each buyer. [Aghion and Bolton (1987, p. 396)].

The type of contracts that Aghion and Bolton consider for the case of two buyers are extended versions of the type of contracts considered for the case of a single buyer. In other words, the price that each buyer must pay if he trades with the incumbent as well as the damages he must pay if he switches to the entrant depend on whether or not the other buyer has already signed a long-term contract.

The incumbent makes simultaneous contract offers to both buyers who play a non-cooperative normal-form game where they have two pure strategies: "accept" and "reject". Aghion and Bolton determine the conditions under which the unique Nash equilibrium is for both buyers to accept the contract.

2. Description of the Model

Inspired by Aghion and Bolton's example of self-enforcing contracts that facilitate entry prevention, in this chapter we will explore the entry-preventing properties of a particular type of contracts between an incumbent firm and two buyers. Before we describe the specific details of this contract, let us start by describing the general idea which motivates the model.

There are a single incumbent, a single potential entrant and two buyers. In period 1, the incumbent spends an exogenously determined amount of money on the R & D of a new product whose production and sales take place in periods 2 and 3 respectively. The potential entrant makes his entry decision in the beginning of period 2 and should he decide to enter, he can enter only at that time. Finally, each buyer wishes to be the single owner of the product.³ If only one buyer purchases and owns one unit of the product then this buyer derives a benefit of R_H . If, however, both buyers purchase one unit of the product, they get a benefit of R_L each with $R_L < R_H$. Buyers can buy one unit of the product either from the incumbent or the entrant if the latter enters.

The incumbent wants to maintain his monopolistic position by preventing entry. One possible way for the incumbent to discourage the potential entrant from entering is to "appropriate" at least one buyer in period 1. This will have the effect of reducing the size of the market that the entrant can serve if he enters. Then if the probability of entry and the size of the market in the post-entry game are positively related, the probability of entry will decrease and entry prevention will be facilitated. The purpose of this paper is to analyze a particular pricing behaviour that can facilitate entry prevention along the above lines.

To arrive at the desired result, two steps must be examined: (1) the incumbent's *ability* and *incentives* to reduce the size of the market in the post-entry game by appropriating at least

one customer in period 1, and (2) given that the incumbent is able and willing to reduce the size of the post-entry market, the impact of such a reduction on the probability of entry. We continue now by examining these steps.

2.1. The Incumbent's Ability to Reduce the Size of the Market in the Post-Entry Game

The incumbent can reduce the size of the market in the post-entry game by offering (simultaneously to both buyers) contracts which are characterized by the following element: if a buyer accepts the contract, this buyer will have to *prepay* the whole amount of the unit price in period 1 while he will be delivered one unit of the product in period 3. If at least one buyer accepts the contract and prepays, the size of the market in the post-entry game will be reduced. It remains to be examined whether such a reduction also reduces the probability of entry (this will be examined below).

Clearly, the incumbent's ability to behave as has just been described depends on the buyers' *willingness* to accept the contract involving prepayments. Both the amount of prepayment and the potential entrant's behaviour influence this willingness. It will be shown that in the complete-information sequential-play game where buyer 1 plays first by choosing between "prepay" and "not prepay" followed by player 2, there can be three subgame-perfect (SP) equilibria. If the expected benefit from prepaying given that the other buyer does not prepay is greater than or equal to the expected benefit from not prepaying given that the other buyer does not prepay as well then there is an incentive for *at least* one buyer to prepay. The number of buyers who actually prepay depends on the relation between the prepayment (denoted by p) and R_L . If the prepayment is greater than R_L and the incumbent is committed not to lower it in subsequent periods, only buyer 1 prepays. If, however, the prepayment is

lower than R_L then both buyers prepay. Finally, if the expected benefit from prepaying given that the other buyer does not prepay is less than the expected benefit from not prepaying given that the other buyer does not prepay as well, the SP equilibrium specifies that neither buyer prepays.

2.2. The Incumbent's Incentives to Reduce the Size of the Market in the Post-Entry Game

It will be shown that in the cases where at least one buyer accepts to prepay, the incumbent's expected profits with prepayments are greater than his expected profits without prepayments. This is going to be true for only some values of the parameters of the model. Nevertheless, these values are consistent with the results that one should expect to get from this model.

2.3. The Size of the Post-Entry Market and the Probability of Entry

It will be shown that if at least one buyer prepays in period 1 and there are sufficiently large fixed costs of entry then the reduction in the size of the post-entry market through the offer of contracts involving prepayments reduces the probability of entry and thus facilitates the entry prevention.

2.4. Efficiency and Ex Ante Welfare

Finally, as far as efficiency and welfare are concerned, comparisons between ex ante welfare with and without prepayments give ambiguous results. In general, prepayments can be welfare-increasing if (1) they prevent the entry of a less efficient producer and/or (2) they

give rise to situations which respect the ranking of the buyers' valuations of owning the product - if, for example, $R_H \gg 2R_L$ then prepayments can be welfare-increasing if they give rise to a situation where a single buyer prepays and gets the product, therefore, deriving a benefit of R_H (provided also that the probability of entry is considerably reduced so that the likelihood that the other buyer will purchase the product from the incumbent is very small).

3. The model

In this section of the paper, we develop in detail a model where prepayments play the role of a barrier to entry. Within this context we explore the motivations for and welfare effects of the use of prepayments.

3.1. Basic Elements of the Model

We consider a three-period model, where an incumbent seller produces a good hoping to supply one unit to each of two buyers. Both buyers are identical and their reservation price for the good depends on whether only one or both purchase the product. If a single buyer purchases the product then his reservation price for one unit of it is R_H while if both buyers purchase the product, each one of them has a reservation price of R_L for one unit of it. It is further assumed that $R_H > R_L$. Buyers can buy one unit of the product either from the incumbent or the entrant if the latter decides to enter.

The incumbent seller has a known and constant unit cost $c_i \in [0,1]$ with $c_i < R_L$ and faces a threat of entry from a potential entrant who considers entering at the beginning of period 2. The entrant's cost of producing the same homogeneous good is not known. For simplicity

we assume that the entrant's cost, c_e , is uniformly distributed in $[0,1]$ and that *only* the entrant learns c_e at the time of entry decision. In addition, the entrant faces a fixed cost of entry, $F > 0$.

If the incumbent and the two buyers knew the entrant's cost then contracts involving prepayments would serve no purpose. If it were common knowledge that the entrant's cost were lower than the incumbent's cost then buyers would not accept any contract in the face of certain entry and a price as low as the incumbent's cost. On the other hand, if it were common knowledge that the incumbent's cost were lower than the entrant's cost then the incumbent would not have to offer contracts in order to foreclose entry.

Hence, the fact that the incumbent and the buyers do not know the entrant's cost induces the incumbent seller to protect his monopoly power by trying to appropriate at least one buyer in period 1. Even though buyers are concerned about the possibility that the entrant's cost is lower than the incumbent's cost, they may stand willing to accept the period-1 contracts in the SP equilibrium of this model. Acceptance may be an equilibrium response (although not necessarily a Pareto optimal one) because if a buyer does not prepay and

- (1) the other buyer has prepaid
- (2) the entrant's cost is higher than the incumbent's cost
- (3) the incumbent seller is committed not to lower the prepayment in subsequent periods

then this buyer will have to do without the product since he will not afford to buy it from the incumbent. Finally, the buyers' acceptance of period-1 contracts on the basis of avoiding the externality that is present whenever a single buyer signs the contract implies that a lower-cost producer may be prevented from entry.

The validity of the above remarks crucially depends on the existence of positive fixed costs of entry. When the entrant must pay a fixed cost of entry then his average cost is decreasing in the number of customers served and the probability of entry is increasing in that number. Thus, even an entrant with a unit cost which is lower than that of the incumbent could be prevented from entry as long as at least one buyer has signed the period-1 contract and fixed costs are sufficiently large. Hence, when the fixed cost of entry can be recovered only through mass production, a buyer's acceptance of the period-1 contract exerts a negative externality on the other buyer.

3.2. Decision Sequence and Information Structure

Prepayment determination results from a sequence of decisions made by the incumbent seller, the entrant and the two buyers. The incumbent initiates this process by selecting a prepayment at which to transfer the product to the buyers. Both buyers observe the prepayment and buyer 1 plays first by choosing between "prepay" and "not prepay". Buyer 2 observes buyer 1's action and then he also decides whether or not to prepay. Prepayments are determined and paid in period 1. Then, the entrant observes his cost, the prepayment and the buyers' decision before he makes his own decision as far as entry is concerned. If both buyers prepay, entry is completely prevented whereas if at most one prepays, entry is possible and can take place in period 2. Finally in period 3, there is trade.

In this context, we are interested in the *ex ante* determination of the prepayment so that we can study the interaction among the prepayment, the size of the potential entrant's market and the probability of entry.

First, we will examine the relation between the size of the potential entrant's market and

the probability of entry (section 3.3) and then the relation between those two and the prepayment (section 3.4) as is revealed from the SP equilibria of the game between the incumbent and the two buyers where the two buyers decide whether or not to accept the contract involving prepayments.

3.3. The Size of the Market in the Post-Entry Game and the Probability of Entry

Let n denote the size of the market left for the entrant to serve in the post-entry game. n can take three values: $n = 0, 1, 2$. If $n = 0$, for instance, then both buyers have prepaid and there is no market left for the entrant. The entrant does not enter and makes zero profits.

If entry occurs, the entrant charges a price of $p_e(n)$ which depends on the size of the potential entrant's market in the post-entry game. If $n = 1$ then buyer 1 prepaies while buyer 2 does not prepay. As we will show below, whenever the equilibrium is characterized by $n = 1$ then it will be true that $p > R_L$. Hence, given that

- (1) buyer 2 has not prepaid
- (2) buyer 2's reservation price, if he purchased the product, would be R_L , and
- (3) the incumbent is committed not to lower the prepayment (p) in period 3,

buyer 2 does not afford to purchase the product from the incumbent in period 3 if entry does not occur. Hence, he can only buy from the entrant who will charge a price of $p_e(1) = R_L$ if he enters. Furthermore, if $n = 2$ (i.e., no prepayments have been paid) and it is profitable for the entrant to enter then the entrant will enter by undercutting the incumbent and charging a price equal to c_e .

Since there is a fixed cost of entry, the entrant will enter if and only if his profits, π_e , are greater than or equal to zero:

$$\pi_e = n[p_e(n) - c_e] - F \geq 0.$$

The probability of entry is given by

$$\alpha(n) = \Pr(\pi_e \geq 0) = \Pr\left[c_e \leq p_e(n) - \frac{F}{n}\right] = p_e(n) - \frac{F}{n}$$

where $\alpha(n) \in [0,1]$.

Given that $p_e(1) = R_L$ and $p_e(2) = c_i$, we have that¹

$$\alpha(1) = R_L - F$$

$$\alpha(2) = c_i - \frac{F}{2}.$$

Then, we have that

$$\alpha(1) < \alpha(2) \quad \text{iff} \quad F > 2(R_L - c_i).$$

This result can be summarized in the following proposition:

Proposition 1. *If the fixed cost of entry, F , and/or the incumbent's unit cost, c_i , are sufficiently large then a decrease in the size, n , of the potential entrant's market reduces the probability of entry.*

As far as c_i is concerned, if c_i is high then the probability of entry when neither buyer has prepaid ($\alpha(2)$) is relatively large because it becomes more likely that the entrant will be more efficient, thus undercutting the incumbent and entering. In the rest of the paper we assume that the parameters of the model are such that proposition 1 is true.

3.4. *Equilibrium Outcomes*

In this section we examine the buyers' incentives to prepay given that the incumbent has already determined the amount of prepayment. We are interested in the interaction between the two buyers in order to be able to determine the number of buyers who will prepay and thus the size of the potential entrant's market as well as how this size varies with the prepayment. This, in turn, will allow us to determine the amount of prepayment that actually reduces the size of the entrant's market and thus facilitates the entry prevention.

The interaction between the two buyers is represented as a two-person, non-cooperative, extensive-form game with complete information (as far as the prepayment is concerned). In any extensive-form game, a player's strategy is a specification of the action it will take in any information set, i.e., the players' actions at any point can depend only on what it knows at that point. Here, the information set for buyer 1 is defined by the prepayment p [and the probabilities $\alpha(1)$ and $\alpha(2)$ of entry]. The same parameters together with buyer 1's choice of action define buyer 2's information set. The possible set of actions that either buyer can select an action from consists of two elements: accept the contract and "Prepay" (P) or reject the contract and, therefore, "Not Prepay" (NP). Let a_i denote the action taken by player i . Then, a pure strategy for 1 is a map s from the possible prepayment levels into $\{P, NP\}$ and a pure strategy for 2 is a map t from all possible pairs (p, a_1) into $\{P, NP\}$.

We will restrict attention to Selten's (1965) criterion of *subgame-perfection*. Formally, the equilibrium of our game is given by the following definition.

Definition 1. A pair of strategies $\sigma^* = (s^*, t^*)$ is a subgame-perfect equilibrium if for every proper subgame the strategy σ^* restricted to the subgame constitutes a Nash equilibrium for the subgame.

The extensive form of the game together with the buyers' expected payoffs is depicted in figure 1.

Given this framework, we can now identify the equilibria of our game.

3.4.1. Case 1: Only Buyer 1 Prepays or $n = 1$

Theorem 1. Suppose that $\alpha(1)R_L + [1 - \alpha(1)]R_H - p \geq \alpha(2)(R_L - c)$ and $p > R_L$. Then the strategy $\sigma_1^* = (s_1^*, t_1^*) = (P, NP)$ is the unique subgame-perfect equilibrium of the game illustrated in figure 1.

Given that buyer 1 plays first, if he chooses *NP* then buyer 2 chooses *P* in which case 1 gets an expected payoff of zero instead of the amount $\alpha(1)R_L + [1 - \alpha(1)]R_H - p > 0$ that he would get if he chose *P* in the first place. Furthermore, he would get this payoff as a result of our assumption that $p > R_L$ which would make buyer 2 to play *NP* (given that buyer 1 had already played *P*). In other words, buyer 1 plays *P* because his expected benefits from prepaying (in which case 2 does not prepay) are greater than his expected benefits from not prepaying (in which case 2 would prepay).

Thus, 1 will accept the contract offered by the incumbent and prepay whereas 2 will not prepay. The incumbent will charge a prepayment equal to $p_1 = \alpha(1)R_L + [1 - \alpha(1)]R_H - \alpha(2)(R_L - c)$ ⁴ and, assuming that he is committed not to lower it in period 3, buyer 2 can purchase one unit of the product from the entrant only whose chances of entering have been reduced by the fact that the size of the post-entry market has been reduced to $n = 1$ as a result of the incumbent's pricing policy. Hence, the incumbent is successful in facilitating the entry prevention by charging a prepayment which is consistent with the inequalities stated in the beginning of theorem 1.

In order to establish that the incumbent will actually use prepayments, we have to show that his expected profits with prepayments are greater than those without prepayments.⁵ The nature of the conditions that determine whether or not prepayments are more profitable than no prepayments depends on the number of buyers that the incumbent seller will serve when his pricing policy does not involve the use of prepayments.

First, if it is more profitable for the incumbent to sell to a single buyer when prepayments do not exist [i.e., $R_H - c_i > 2(R_L - c_i)$] then prepayments will yield more expected profits than no prepayments if and only if $\alpha(1) < \alpha(2)$, i.e., if and only if the fixed cost of entry is sufficiently large or, in other words, if and only if the reduction in the size of the potential entrant's market reduces considerably the probability of entry thus facilitating the entry prevention. That is, if the incumbent finds it profitable to sell to a single buyer either with or without prepayments, he will prefer prepayments to no prepayments only if they weaken or eliminate the threat of entry.

Second, if it is more profitable to sell to both buyers rather than only one when prepayments do not exist, one needs to compare the expected profits from selling to buyer 1 only (in the case of prepayments) with the expected profits from selling to both buyers (in

the case where prepayments do not exist). The incumbent's expected profits from supplying one unit to each of the two buyers (when his pricing policy does not involve the use of prepayments) are reduced by $2\alpha(2)(R_L - c_i)$ which describes the decrease in the incumbent's expected profits because of the ex ante positive probability that a more efficient entrant will enter and serve the entire market. Then, the incumbent seller will consider switching to a pricing policy with prepayments that reduce the size of the entrant's market in the post-entry game to $n = 1$ if (a) the probability of entry $[\alpha(1)]$ is sufficiently small; (b) $2\alpha(2)(R_L - c_i)$ is sufficiently large; and (c) R_H is sufficiently larger than R_L . The last point implies that each buyer is prepared to pay a high prepayment in order to become the single owner of the product. Then, the incumbent seller has the incentive to allow only one buyer to prepay by charging a prepayment greater than R_L and thus reducing the entrant's market in the post-entry game to $n = 1$.

An Example. Suppose that $c_i = 0.8$, $F = 0.9$ and $R_L = 1$. Then, we have that $\alpha(1) = R_L - F = 0.1$ and $\alpha(2) = c_i - F/2 = 0.35$. Furthermore, let us assume that R_H satisfies the following inequality:

$$R_H > R_L + \frac{\alpha(2)}{1 - \alpha(1)}(R_L - c_i) = 1.078$$

and let R_H be equal to 2. Note that $R_H - c_i = 1.2 > 2(R_L - c_i) = 0.4$ so that selling to a single buyer when prepayments do not exist is more profitable than selling to both buyers.

Now, we can calculate the prepayment which is equal to $p_i = \alpha(1)R_L + [1 - \alpha(1)]R_H - \alpha(2)(R_L - c_i) = 1.83$. Notice that $p_i > R_L$ so that theorem 1 is true, implying that only buyer 1 prepays. Moreover, the incumbent's expected profits with prepayments are greater than

those without prepayments:

$$E\pi_{L2} = p_1 - c_L = 1.03 > E\pi_{L2} = [1 - \alpha(2)](R_H - c_1) = 0.78.$$

3.4.2. Case 2: Both Buyers Prepay or $n = 0$

Theorem 2. Suppose that $\alpha(1)R_L + [1 - \alpha(1)]R_H - p \geq \alpha(2)(R_L - c_1)$ and $p < R_L$. Then the strategy $\sigma_2^* = (s_2^*, t_2^*) = (P, P)$ is the unique subgame-perfect equilibrium of the game depicted in figure 1.

Hence, under these parameter specifications both buyers choose to prepay in equilibrium. If buyer 1 did not prepay then buyer 2 would prepay in which case 1 would get zero instead of $R_L - p > 0$ he would get if he prepaid. In other words, since 2 prepaies irrespective of what 1 does, 1 finds it optimal to prepay as well. Thus, 1 will accept the contract and prepay and then 2 will prepay as well. Furthermore, since both players prepay there is no market left for the entrant. Entry is completely prevented.

In this case the prepayment is equal to $p_0 = \alpha(1)R_L^* + [1 - \alpha(1)]R_{H0} - \alpha(2)(R_L - c_1)$. p_0 will be less than R_L if and only if R_{H0} is sufficiently lower than R_L (as was explained in note 4):

$$R_{H0} < R_L + \frac{\alpha(2)}{1 - \alpha(1)}(R_L - c_1).$$

Furthermore, since $p_1 > R_L$ and $p_0 < R_L$, we have that $p_0 < p_1$. That is, the equilibrium prepayment when both buyers prepay is less than the equilibrium prepayment for the case where only one buyer prepaies.

In light of the fact that both buyers prepay in equilibrium, it is interesting to point out that each buyer's expected benefit from prepaying given that both prepay is less than his expected benefit when both buyers reject the period-1 contracts and do not prepay, i.e., $R_L - p_0 < \alpha(2)(R_L - c)$.⁶ As a result, total consumer surplus is maximized when both buyers reject the contract offered by the incumbent and do not prepay. Nevertheless, the equilibrium behaviour of each buyer imposes a negative externality on the other buyer. By locking himself into a contractual relation with the seller, he reduces the size of the potential entrant's market so that the probability of entry will be reduced. Then, given that $p_0 < R_L$ which implies that both buyers afford to buy from the incumbent, the other buyer prefers to buy from the incumbent as well rather than face the prospect of no entry which would force him to get the product from the incumbent paying R_L .

As in case 1, the nature of the conditions that render prepayments more profitable than no prepayments depends on the number of customers that the incumbent seller serves when prepayments do not exist. First, if, without prepayments, selling to a single buyer is more profitable than selling to both then expected profits with prepayments are greater than those without prepayments if and only if $\alpha(1) < \alpha(2)$, i.e., if and only if prepayments reduce the probability of entry. This is obvious for the present case since both buyers prepay leaving no market for the entrant to serve in the post-entry game.

Second, if selling to a single buyer is less profitable than selling to both then we should compare expected profits from selling to both buyers when prepayments are used with expected profits from selling to both buyers when prepayments do not exist. The maximal additional expected profits that the incumbent seller can hope to earn by switching from a pricing policy involving prepayments to one without prepayments are equal to $2[1 - \alpha(2)](R_L - p_0)$ and corresponds to the case where there is no entry. On the other hand, the incumbent

seller can expect to lose $2\alpha(2)(p_0 - c_i)$ if he switches pricing policies. That is, if he abandons prepayments then entry may take place with probability $\alpha(2)$ in which case both buyers purchase the product from the entrant and thus depriving the incumbent of the profits that he would have made if he had not switched policies. Hence, if $2\alpha(2)(p_0 - c_i) > 2[1 - \alpha(2)](R_L - p_0)$, expected profits from selling to both buyers when prepayments are implemented are larger than expected profits from selling to both buyers when prepayments are not in use.

An Example. We can use the same example as in case 3.4.1 in order to illustrate the above. Instead of choosing $R_H > 1.078$, let's choose a value for R_H less than 1.078. Suppose that $R_H = 1.05$. Then, the prepayment is equal to $p_0 = 0.975$. Since $p_0 < R_L$, theorem 3.2 applies so that both buyers prepay. Note also that $R_H - c_i = 0.25 < 2(R_L - c_i) = 0.4$. Finally, prepayments are more profitable than no prepayments since

$$E\pi_{pp} = 2(p_0 - c_i) = 0.35 > E\pi_a = 2[1 - \alpha(2)](R_L - c_i) = 0.26.$$

3.4.3. Case 3: No Buyer Prepays or $n = 2$

Theorem 3. Suppose that $\alpha(1)R_L + [1 - \alpha(1)]R_H - p < \alpha(2)(R_L - c_i)$. Then the strategy $\sigma_j^* = (s_j^*, t_j^*) = (NP, NP)$ is the unique subgame-perfect equilibrium of the game depicted in figure 1.

The inequality in the theorem implies that each buyer's expected benefits from prepaying given that the other buyer does not prepay are smaller than his expected benefits from not prepaying given that the second buyer does not prepay as well. It is also true that $R_L - p <$

$\alpha(1)R_L + [1 - \alpha(1)]R_H - p$ and, therefore, $R_L - p < \alpha(2)(R_L - c_i)$. This means that each buyer's expected benefits from prepaying given that the second buyer prepay as well are less than his expected benefits from not prepaying given that the other buyer does not prepay.

As a result, each buyer's expected benefits from prepaying (regardless of whether or not the other buyer prepay as well) are smaller than his expected benefits from not prepaying given that the other buyer does not prepay. This induces both buyers to reject period-1 contracts that involve prepayments.

The situation, therefore, is the same as if no prepayments were charged in the first place. Under these circumstances, as soon as the entrant observes his cost in the beginning of period 2, he will enter if his profits from doing so are greater than or equal to zero; that is, if

$$2[p_s(2) - c_s] - F \geq 0$$

or, equivalently, if

$$c_s \leq p_s(2) - \frac{F}{2} - c_i - \frac{F}{2}$$

Hence, the entrant will enter with probability $c_i - (F/2)$ in which case he will undercut the incumbent and charge a price equal to c_i .

Finally, the SP equilibrium strategy (NP,NP) is also Pareto optimal since the buyers' expected consumer surplus is maximized when both buyers do not accept to prepay. The reason why both buyers play the Pareto-optimal strategy is that there is no negative externality involved which would induce one buyer to prepay for fear that if he did not, the other buyer would prepay leaving the former with the prospect of doing without the product

or purchasing the product from the incumbent paying a higher period-3 price. Both buyers know that prepaying is not in the interest of either one regardless of the action each one takes. Hence, both play *NP*.

4. Expected Profits Versus Expected Consumer Surplus

An interesting observation is that prepayments enable the incumbent to extract large surpluses from the individual buyers, which he could not do if prepayments did not exist or if the buyers colluded with the purpose of rejecting prepayments.

To see why, let us compare the change in the incumbent's expected profits with the change in the expected consumer surplus in two different situations. First, as we move from a pricing policy without prepayments to one with prepayments which completely prevent entry and, second, to one with prepayments reducing the size of the potential entrant's market to $n = 1$. The following propositions summarize our results.⁷

Proposition 2. *The increase in the incumbent's expected profits brought about by a change in his pricing policy from a regime without prepayments to one with prepayments that completely prevent entry is equal to the decrease in the expected consumer surplus caused by the same change.*

Proposition 3. *The increase in the incumbent's expected profits brought about by a change in his pricing policy from a regime without prepayments to one with prepayments that reduce the potential entrant's market in the post-entry game to $n = 1$ is less than the decrease in the expected consumer surplus caused by that change.*

The reduction in the expected consumer surplus caused by the incumbent's adoption of a pricing policy involving prepayments has its origins in the negative externality that prompts buyers to prepay in the SP equilibrium although such a strategy is not Pareto optimal. Proposition 2 suggests that this reduction is translated to an equal-sum increase in the incumbent's expected profits for the case where prepayments completely prevent entry.

Proposition 3, as opposed to proposition 2, says that the amount by which the expected consumer surplus decreases because of the use of prepayments (that reduce the entrant's market in the post-entry game to $n = 1$) is only partly appropriated by the incumbent. The reason is that only buyer 1 prepays whereas 2 buys from the entrant if he enters.

5. $n = 1$ Versus $n = 0$: Relative Profitability of Prepayments

Prepayments that prevent entry completely are not necessarily more profitable than those that just reduce the probability of entry. It may be optimal for the incumbent not to eliminate entry completely. The following proposition summarizes the relevant results.

Proposition 4. *If $R_{H1} \geq 2R_{H0}$ then prepayments that reduce the size of the post-entry market to $n = 1$ yield more expected profits than prepayments which prevent entry completely.*

Proof. From appendix A2 we have that

$$E\pi_U = p_1 - c_1$$

$$E\pi_B = 2(p_0 - c_i)$$

where $p_i = \alpha(1)R_L + [1 - \alpha(1)]R_{H1} - \alpha(2)(R_L - c_i)$ and $p_0 = \alpha(1)R_L + [1 - \alpha(1)]R_{H0} - \alpha(2)(R_L - c_i)$. Then, by substituting for p_i and p_0 and going through the necessary algebra, we get the following expression which explains proposition 4.

$$E\pi_H - E\pi_B = [1 - \alpha(1)](R_{H1} - 2R_{H0}) - [\alpha(1) - \alpha(2)]R_L + [1 - \alpha(2)]c_i$$

Intuitively, a relatively high R_{H1} enables the incumbent seller to charge a high prepayment since each buyer is willing to pay a large amount in order to obtain exclusivity on the use of the product. Thus, prepayments that result in a lock-in of just a single buyer may yield high expected profits even though they only reduce the probability of entry and appropriate only part of the decrease in expected consumer surplus (as proposition 3 suggests).

6. Welfare Effects of Prepayments

The impact of prepayments on ex ante welfare (defined as the unweighted sum of expected profits and expected consumer surplus) depends on the values of the parameters of the model so that comparisons between expected welfare with and without prepayments give rise to ambiguous results.⁸ Nevertheless, the following remarks can be made.

1. In situations where the potential entrant's unit cost is higher than that of the incumbent, prepayments can be welfare-increasing since they facilitate the entry prevention of a less efficient producer. Note, however, that prepayments may reduce the probability of entry of a *more* efficient producer. This can happen in the case where only buyer 1 prepay so that

the size of the potential entrant's market is $n = 1$. Then the entrant will enter if his unit cost is lower than R_L . Since we have assumed that $R_L > c$, it may be the case that c is higher than the entrant's unit cost so that prepayments can reduce the probability of entry of a more efficient producer.

2. Prepayments can be welfare-increasing if they give rise to ownership situations that respect the ranking of the buyers' valuations of owning the product. To make this point clear, suppose, for example, that R_H is high relatively to R_L . Then the value derived from a situation where a single buyer owns the product is high compared to the value of a situation where both buyers buy the product. Since prepayments facilitate the entry prevention of the potential entrant whenever $n = 1$, they increase the probability that buyer 1 will be the *single* owner of the product so that a value of $R_H \gg R_L$ rather than R_L will be realized. Nevertheless, whenever the incumbent's product is an intermediate input, one has to take into account any repercussions that such a development may have for the downstream final-product market. More specifically, if a monopoly is established in the final-product market as a consequence of a single buyer purchasing the intermediate input from the incumbent, a welfare loss is created which has to be taken into account.

In order to assess the impact of prepayments on *ex ante* welfare, we should take into consideration all of the above factors.

7. Conclusion

Pricing strategies that involve the use of prepayments can be thought of as being part of a wider range of agreements amongst established firms and customers that limit access by potential competitors to particular markets. Prepayments, therefore, may have considerable

applicability.

It has been argued by many economists that contracts between buyers and sellers in intermediate-good industries may have anticompetitive entry-prevention effects and that such contracts may be welfare-decreasing. On the other hand, many others have pointed out that contracts between buyers and sellers are socially efficient. Bork (1978) and Posner (1976), for example, have made the point that even though buyers are better off when there is entry and increased competition, they tend to accept entry-preventing contracts if the seller compensates them for the adverse effects of no entry. In our model, however, buyers accept entry-preventing contracts even though the acceptance of prepayments emerges as a SP equilibrium which is *not* optimal from the buyers' point of view. In other words, total expected consumer surplus is maximized by the rejection rather than the acceptance of prepayments. This, in turn, implies that the incumbent's expected profits increase at the expense of both the buyers' expected consumer surplus and the potential entrant's expected profits.

There is an analogy to be drawn between the present model and Klemperer's (1987) model on entry deterrence in markets with switching costs. Klemperer examines how the threat of new entry affects an incumbent's behaviour in a market with switching costs and thus provides an explanation of limit pricing behaviour. An incumbent monopolist threatened by entry may underprice and thus overproduce before the date in which the entrant makes his entry decision in order to attract more customers and thus reduce the size of the market that the entrant would serve if he entered. Although Klemperer's model shares the previous idea with ours, the details of the analysis are different. Klemperer considers a market which operates under conditions of complete and perfect information and in which each consumer incurs an exogenous switching cost of buying either from the incumbent or the entrant.

Furthermore, if entry occurs then there is a Cournot equilibrium in period 2 which implies that period 1 overproduction by the incumbent reduces the entrant's profit in period 2, given that the incumbent's period-1 customers have to incur a cost if they want to switch to the entrant. The incumbent may overproduce to the point where the entrant's profits become smaller than the fixed cost of entry.

Our model, however, is formulated in terms of *serving capacity* rather than *production capacity*: the firm chooses a price and a number of customers to be served rather than a price and a quantity to be produced. This implies that there is no overproduction since if the monopolist were unthreatened by entry, he would serve both buyers. Entry is prevented by the combination of two factors: the presence of a negative externality whenever only one buyer prepayes and the fact that the entrant's average cost is decreasing in the number of consumers served.

Appendix

A1. Proof of the Conditions that Render Prepayments More Profitable than No Prepayments for the Case of $n = 1$

Case 1. Selling to a Single Buyer is More Profitable than Selling to Both When Prepayments Are Not in Use: $R_{H1} - c_i > 2(R_L - c_i)$

We want to show that $E\pi_1 > E\pi_2$ or, equivalently,

$$p_i - c_i > [1 - \alpha(2)](R_{H1} - c_i).$$

Substituting for p_i , the above inequality can be written as follows:

$$\begin{aligned} \alpha(1)R_L + [1 - \alpha(1)]R_{H1} - \alpha(2)(R_L - c_i) - c_i &> [1 - \alpha(2)](R_{H1} - c_i) \\ \Rightarrow \alpha(1)R_L + [1 - \alpha(1)]R_{H1} - \alpha(2)R_L &> [1 - \alpha(2)]R_{H1} \\ \Rightarrow [\alpha(1) - \alpha(2)](R_L - R_{H1}) &> 0. \end{aligned}$$

Since $R_{H1} > R_L$, $E\pi_1 > E\pi_2$ if and only if $\alpha(1) < \alpha(2)$.

Case 2. Selling to Both Buyers is More Profitable than Selling to Only One When Prepayments Are Not in Use: $R_{H1} - c_i < 2(R_L - c_i)$

We want to show that

$$p_1 - c_i > 2[1 - \alpha(2)](R_L - c_i).$$

Substituting for p_i we get

$$\alpha(1)R_L + [1 - \alpha(1)]R_{HI} - \alpha(2)(R_L - c_i) - c_i > 2[1 - \alpha(2)](R_L - c_i)$$

$$\Rightarrow \alpha(1)R_L - 2R_L + [1 - \alpha(1)]R_{HI} + \alpha(2)(R_L - c_i) + c_i > 0$$

$$\Rightarrow \alpha(1)(R_L - R_{HI}) + (R_{HI} + c_i - 2R_L) + \alpha(2)(R_L - c_i) > 0.$$

The discussion in section 3.4.1 is based on this inequality.

A2. Propositions 2 and 3: Some Steps of the Proof

It only takes some algebra to show that these propositions are true. The expressions for the incumbent's expected profits and the buyers' expected consumer surplus are given by the following.

(i) $n = 2$.

$$\begin{aligned} E\pi_2 &= 2[1 - \alpha(2)](R_L - c_i) \\ ECS_2 &= 2\alpha(2)(R_L - c_i). \end{aligned}$$

(ii) $n = 1$.

$$\begin{aligned} E\pi_1 &= p_1 - c_i \\ ECS_1 &= \alpha(1)R_L + [1 - \alpha(1)]R_{HI} - p_1. \end{aligned}$$

(iii) $n = 0$.

$$\begin{aligned} E\pi_0 &= 2(p_0 - c_i) \\ ECS_0 &= 2(R_1 - p_0). \end{aligned}$$

Then, propositions 2 and 3 can be stated alternatively as follows.

$$\begin{aligned} E\pi_0 - E\pi_1 &= ECS_2 - ECS_0 \\ E\pi_1 - E\pi_2 &< ECS_2 - ECS_1. \end{aligned}$$

A3. Welfare Effects of Prepayments

In this section we compare between expected welfare with and without prepayments and we derive the expressions that can be used to explain the remarks that have been made in section 6 of the paper. We start with a situation without prepayments by calculating ex post welfare with entry (W_E) and with no entry (W_N). Then, we calculate ex ante welfare by integrating ex post welfare over the relevant values of the potential entrant's unit cost, c_e . We will follow the same procedure for the cases with prepayments ($n = 1$ and $n = 0$). Finally, we will restrict attention to the case where it is more profitable for the incumbent to sell to a single rather than both buyers when prepayments are not used.

(1) No Prepayments

$$W_N = (R_N - c_i) + (R_N - R_N) = R_N - c_i$$

$$W_E = 2(c_i - c_s) - F + 2(R_L - c_i) - 2(R_L - c_s) - F.$$

Ex ante welfare is equal to the following:

$$\begin{aligned} EW &= \int_0^{c_i - F/2} W_E dc_s + \int_{c_i - F/2}^1 W_N dc_s \\ &= \int_0^{a(2)} [2(R_L - c_s) - F] dc_s + \int_{a(2)}^1 (R_N - c_i) dc_s \\ &= a(2)(2R_L - F) - \frac{1}{2} [a(2)]^2 + [1 - a(2)](R_N - c_i). \end{aligned}$$

(ii) Prepayments: $n = 1$

$$\begin{aligned} W_{N1} &= (p_1 - c_i) + (R_{N1} - p_1) - R_{N1} - c_i \\ W_{E1} &= (p_1 - c_i) + (R_L - c_s - F) + (R_L - p_1) + (R_L - R_L) - 2R_L - c_i - c_s - F \\ EW_1 &= \int_0^{R_L - F} W_{E1} dc_s + \int_{R_L - F}^1 W_{N1} dc_s \\ &= \int_0^{a(1)} (2R_L - c_i - c_s - F) dc_s + \int_{a(1)}^1 (R_{N1} - c_i) dc_s \\ &= a(1)(2R_L - c_i - F) - \frac{1}{2} [a(1)]^2 + [1 - a(1)](R_{N1} - c_i). \end{aligned}$$

(iii) Prepayments: $n = 0$

Since both buyers prepay, the incumbent's pricing strategy prevents entry completely. Expected welfare is therefore equal to the following:

$$EW_0 = 2(p_0 - c_i) + 2(R_L - p_0) = 2(R_L - c_i).$$

(iv) Comparisons

$$\blacksquare EW - EW_i$$

$$EW - EW_i = [\alpha(2) - \alpha(1)](2R_L - F) - [\alpha(2) - \alpha(1)](R_{HI} - c_i) + \alpha(1)c_i + \frac{1}{2}[\alpha(1)^2 - \alpha(2)^2]$$

$$= [\alpha(2) - \alpha(1)](2R_L - R_{HI} - F) + \alpha(2)[c_i - \alpha(2)] + \frac{1}{2}[\alpha(1)^2]$$

$$= [\alpha(2) - \alpha(1)](2R_L - R_{HI} - F) + \frac{F}{2}\alpha(2) + \frac{1}{2}[\alpha(1)^2].$$

*

Hence, a pricing policy that involves a prepayment equal to p_i may increase expected welfare if R_{HI} and/or F are sufficiently high.

$$\blacksquare EW - EW_0$$

$$EW - EW_0 = \alpha(2)(2R_L - F) - [\alpha(2)]^2 + [1 - \alpha(2)](R_{HI} - c_i) - 2(R_L - c_i).$$

A pricing policy that involves prepayments which prevent entry completely may increase expected welfare if $2(R_1 - c_1)$ and/or F are sufficiently high.

Notes

1. Gilbert (1981) and Gilbert and Newbery (1982) consider the implications of patent activity for increasing the cost of entry by denying access to the technology by competing firms as well as for improving productivity.

2. This assumption can be motivated by thinking of each buyer as being a downstream firm and of the incumbent's product as being a key intermediate input. If only one downstream firm purchases and uses that input then this firm can exercise monopoly power in the final-good market.

3. The requirement that $\alpha(n) \in [0, 1]$ implies some constraints for the value of F in relation to R_L and c . Using the equations for $\alpha(1)$ and $\alpha(2)$, these constraints are the following:

$$\begin{aligned} R_L - 1 &\leq F \leq R_L \\ 2(c_L - 1) &\leq F \leq 2c_L \end{aligned}$$

Furthermore, there exist values of F that satisfy *both* the above inequalities *and* the requirement that $F > 2(R_L - c)$.

4. The reason why we have put a subscript in R_H is that we will assume that different equilibria and prepayment levels arise because of variations in R_H . Indeed, p can be greater than R_L if and only if R_H is sufficiently greater than R_L :

$$p > R_L \Leftrightarrow R_H > R_L + \frac{\alpha(2)}{1 - \alpha(1)} (R_L - c)$$

where $p = \alpha(1)R_L + [1 - \alpha(1)]R_H - \alpha(2)(R_L - c)$.

Hence, any R_H , given the value of R_L , that satisfies the above inequality (we assume that R_H does) gives rise to a level of prepayment which is greater than R_L and which reduces the size of the potential entrant's market to $n = 1$. Intuitively, the higher R_H above R_L , the greater the benefit from being the single owner of the product (relative to the benefit derived from a situation where both buyers purchase the product) and the larger the payment that buyer 1 is prepared to pay in order to obtain exclusivity on the use of the product. This implies that the incumbent's degrees of freedom to charge a larger prepayment increase and if R_H becomes sufficiently high, the incumbent's expected profits from selling to a single buyer become greater than those from selling to both buyers. Then, the incumbent has the incentive to prevent the second buyer from prepaying and he can do that by charging a prepayment equal to $p > R_L$. Hence, if the value of R_H is such that the above inequality is true then $p > R_L$ and we obtain theorem 1.

5. In appendix A1 we work out the difference between the expected profits with and without prepayments and we derive the expressions that the discussion in section 3.4.1 is based on.

6. This can be easily shown by substituting for p_0 .

7. Some steps of the proof of these propositions are presented in appendix A2.

8. Appendix A3 provides some expressions that help to make these comparisons.

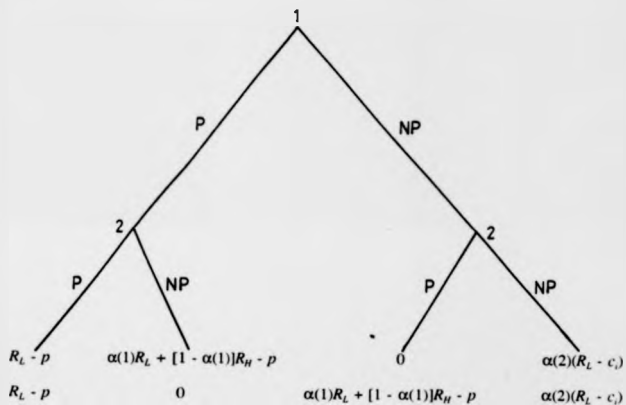


Figure 1. *The Extensive-Form Game Between Buyers 1 and 2*

PART 3

CONCLUSION

By way of conclusion, we will focus our discussion on (1) the attitudes of the market towards the "real" (as opposed to financial) risk involved in the process of innovation and (2) how this issue can be addressed by an extended version of chapter 3. We find this discussion to be very interesting for reasons that will become apparent below.

Whether or not the market is biased against risky R & D has been an issue that has been investigated by many authors. Dasgupta and Stiglitz (1980) argue that even if firms are risk neutral, the market is biased against more risky research programmes compared to a social planner. On the other hand, Klette and de Meza (1986) argue that the market will adopt a relatively safe research strategy only if there are few competitors in the market. Pavitt (1976), in discussing the rationale for government intervention in the process of innovation, argues that since research is a long-term and risky activity, firms will tend to underinvest in technological change and concentrate their research efforts on short-term improvement innovations.

The basic argument, however, of the above discussion dates back to Arrow (1962). Arrow first examines an ideal economy in which the problem of allocating risk is solved by competition and then indicates some of the instruments that are actually available for shifting risk. In Arrow's model, firms make input decisions but there is uncertainty as far as the output that is generated by these inputs is concerned. In other words, the outputs are determined by the inputs and an unknown "state of nature". The production of a given

commodity under uncertainty can then be described as the production of a vector of "commodity-options", where a commodity-option is a commodity in the ordinary sense labelled with a state of nature. If markets for all commodity-options existed then agents would trade contracts in each market with each contract requiring that the buyers should pay an agreed sum and that the seller should deliver a specified quantity of a given commodity if a certain state of nature prevails and nothing if that state of nature does not occur. In this ideal economy, the markets for commodity-options would achieve an optimal allocation of risk among the economy's agents. Risk bearing and production would be treated separately so that "the use of inputs in their most productive mode would not be inhibited by unwillingness or inability to bear risks by either firms or productive agents" [Arrow (1962, p. 611)]. Nevertheless, markets for commodity-options do not exist which implies that firms cannot relieve themselves of risk bearing. An inability to shift risk will generate an underinvestment in risky enterprises as compared with the optimum.

Going back to chapter 3, it is clear that R & D spending for a new product is a risky activity simply because, as Stoneman (1987) notes, it is

largely the exploration of a new territory and the outcome of any particular project in technical terms cannot be known with certainty. Moreover the profit to be made from technical advance cannot be known with certainty-the return will depend *inter alia* on the discoveries made by other firms, consumer reactions, and in general the possibilities of appropriating the benefits. [Stoneman (1987, p. 118)].

As Arrow points out, there are a number of mechanisms available for shifting risk, such as insurance policies and cost-plus contracts. Problems, however, still exist since these risk-shifting instruments may have the effect of dulling incentives and they cover only a small range of events in the real world. A third possibility is that it can be optimal for the

government to carry the burden of some private-sector risk if the government has a superior ability to carry risk or is less risk averse than individuals [Stoneman (1987, p. 118)].

The model in chapter 3 (extended to the case where the amount of money spent on R & D is an endogenous variable) offers an alternative risk-shifting mechanism that can alleviate the problem of possible underinvestment. In effect, this mechanism amounts to the creation of markets for Arrow's commodity-options. In our model, these markets take a simple form. Since the monopolist determines his expected profit-maximizing pricing policy before the actual value of the unit cost is realized, there are two states of nature: either the unit cost is revealed to be greater than the period-1 price in which case the commodity is not produced or is revealed to be less than or equal to the period-1 price in which case the commodity is produced. Hence, there are two commodity-options and there are also markets for both of these options. Period-1 customers agree to pay a period-1 unit price and the seller agrees to deliver one unit of the product in period 1 if the unit cost turns out to be less than or equal to that price. They also agree to pay a prepayment and the seller agrees to deliver no quantity in any other state of nature.

A particular example of an environment that can be used to analyze the issue of risk-shifting along the above lines is one where the producer's investment on R & D directly affects the reliability of the production process in terms of the delivery time. If there is no risk-shifting mechanism, the producer will underinvest on the reliability of the production process which thus may result to frequent breakdowns and delays in the delivery of the product. Prepayments can be a risk-shifting mechanism if buyers have different valuations of the good and prefer to have the good earlier rather than later. Then, the higher the valuation of a buyer, the greater the risk that this buyer is willing to carry and the greater the prepayment that he is willing to pay as long as a greater prepayment implies that a lower risk

is carried by the producer.

The issue of risk-shifting in the context of chapter 3 is an interesting one and is one that we hope to investigate in detail.

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